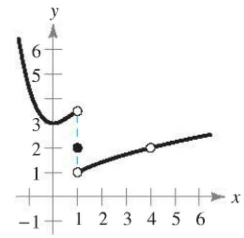
Math 2401 Final exam review

1. Use the graph of the function f to decide whether the value of the given quantity exists. If it does, find it. If not, explain why.



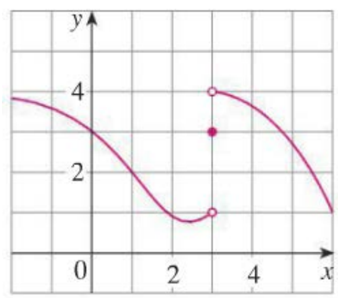
$f(1)$=_____2_____.

$\lim_{x\to 1} f(x)$=___DNE_____.

$f(4)$=____DNE______.

$\lim_{x\to 4} f(x)$=_2_____.

2. For the function f whose graph is given, state the value of each quantity, if it exists. If it does not exist, use DNE.



a. $\lim_{x\to 1} f(x)$ b. $\lim_{x\to 3^-} f(x)$ $\lim_{x\to 3^+} f(x)$

Solution:

a. $\lim_{x\to 1} f(x) = 2$

b. $\lim_{x\to 3^-} f(x) = 1$

c. $\lim_{x\to 3^+} f(x) = 4$

3. Find the limit of the function.

a. $\lim_{x\to 0} \frac{x^4-5x^2}{x^2}$. $= -5$

b. $\lim_{x\to -1} \frac{x^2-2x-3}{x+1}$ $= -4$

c. $\lim_{x\to 1} \frac{x^2-x}{x^2-5x+4}$ $= -\frac{1}{3}$

d. $\lim_{x\to 2} \frac{x^2-2x}{x^2-4}$ $= \frac{1}{2}$

4. Find the derivative of the function.

a. $f(x) = (3x^2 - 4)(2x + 5)$

$f'(x) = 6x\,(2x + 5) + (3x^2 - 4)2 = 18x^2 + 30x - 8$.

b. $g(s) = \sqrt{s}\,(s^2 + 8)$

$$g'(s) = \frac{1}{2}s^{-\frac{1}{2}}(s^2+8) + s^{\frac{1}{2}}\,2s$$

$$= \frac{1}{2}s^{\frac{3}{2}} + 4s^{-\frac{1}{2}} + 2s^{\frac{3}{2}}$$

$$= \frac{5}{2}s^{\frac{3}{2}} + 4s^{-\frac{1}{2}} \quad or \quad \frac{5}{2}s^{\frac{3}{2}} + \frac{4\sqrt{s}}{s}$$

c. $h(t) = t^3 \cos t$

$$h'(t) = 3t^2 \cos t + t^3\,(-\sin t)$$
$$= \boxed{3t^2 \cos t - t^3 \sin t}$$

d. $f(x) = \frac{\sqrt{x}}{x^2+1}$

$$f'(x) = \frac{\left(\frac{1}{2}x^{-\frac{1}{2}}(x^2+1) - x^{\frac{1}{2}}(2x)\right)}{(x^2+1)^2}$$
$$= \frac{\frac{1}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{-\frac{1}{2}} - 2x^{\frac{3}{2}}}{(x^2+1)^2}$$
$$= \frac{\left(-\frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\right)}{(x^2+1)^2}$$
$$= \boxed{\frac{x^{\frac{1}{2}} - 3x^{\frac{5}{2}}}{2x(x^2+1)^2}}$$

e. $g(x) = \frac{sinx}{x^2}$

$$g'(x) = \frac{x^2 \cos x - 2x \sin x}{x^4}$$
$$= \boxed{\frac{x \cos x - 2 \sin x}{x^3}}$$

5. Find $f'(x)$ and find the tangent line of $f(x)$ at the giving point c.

a. $f(x) = (x^2 + 4x)(3x^2 + 2x - 5)\,,\ c = 0$

$$f'(x) = (2x+4)(3x^2+2x-5) + (x^2+4x)(6x+2)$$
$$= \boxed{12x^3 + 42x^2 + 6x - 20}$$

Tangent line: $\boxed{y = -20\,x}$

b. $f(x) = x\cos(x),\ c = \frac{\pi}{4}$

$$f'(x) = \cos x + x\,(-\sin x)\ = \cos x\ - x \sin x$$

Tangent line: $y = \frac{4\sqrt{2} - \pi\sqrt{2}}{8}x + \frac{\pi^2\sqrt{2}}{32}$

c. $f(x) = \frac{x-4}{x+4}, c = 3$

$$f'(x) = \frac{8}{(x+4)^2}$$

Tangent line: $y = \frac{8}{49}x - \frac{31}{49}$

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6. Find the derivative of the trigonometric function.

a. $f(t) = t^2 \sin t$

$$f'(t) = 2t \sin t + t^2 \cos t$$

b. $f(\theta) = (\theta^2 + 1)\cos\theta$

$$f'(\theta) = 2\theta\cos\theta - (\theta^2 + 1)\sin\theta$$
$$= 2\theta\cos\theta - \theta^2\sin\theta - \sin\theta$$

c. $y = \frac{\sec x}{x}$

$$\frac{dy}{dx} = \frac{\tan x \sec x - \sec x}{x^2}$$

d. $f(x) = x^2 \tan x$

$$f'(x) = 2x\tan x + x^2\sec^2 x$$

e. $g(x) = \frac{\sin\theta}{1-\cos\theta}$

$$g'(x) = \frac{\cos\theta - 1}{(1-\cos\theta)^2}$$

7. Find the second derivative of the function.

a. $f(x) = x^4 + 2x^3 - 3x^2 - x$

$$f''(x) = 12\,x^2 + 12x - 6$$

b. $f(x) = \sec x$

$$f''(x) = \sec^3 x + \tan^2 x \sec x$$

8. Find the derivative of the following functions.

a. $y = \sqrt{4 - 3x^2}$

b. $f(x) = \sqrt{x^2 - 4x + 2}$

$\frac{dy}{dx} = -3x\,(4 - 3x^2)^{-\frac{1}{2}}.$

$f'(x) = (x-2)(x^2 - 4x + 2)^{-\frac{1}{2}}$

c. $y = \sin(2x)\cos(2x)$

$y' = 2\cos^2 2x - 2\sin^2 2x$

d. $y = x^2 \sec x^2$

$y' = 2x\sec x^2 + 2x^3\tan x^2 \sec x^2$

e. $y = \sin(\pi x)^2$

$y' = 2\,\pi^2 x\cos(\pi x)^2$

f. $h(x) = 5\tan(3x)$

$h'(x) = 15\sec^2(3x)$

9. Use implicit differentiation to find y' and evaluate y' at the indicated point.

a. $y^2 - y - 4x = 0;\ \ (0,1)$

$y'|_{(0,1)} = 4$.

b. $3xy - 2x - 2 = 0;\ \ (2,1)$

$y'|_{(2,1)} = -\frac{1}{6}$

c. $2xy + y + 2 = 0;\ \ (-1,2)$

$y'|_{(-1,2)} = 4$

10. For the given function $f(x) = (x-1)^2\,(x+3)$

a.Find the critical numbers.

$c = 1, -\frac{5}{3}$

b.Find the open intervals on which the function is increasing or decreasing.

Interval	$\left(-\infty, -\frac{5}{3}\right)$	$\left(-\frac{5}{3}, 1\right)$	$(1, \infty)$
Test point	-3	0	2
$f'(x)$ +/- at the test point	+	-	+
Increasing or decreasing	Increasing	Decreasing	increasing

c.Find the inflection points.

Inflection point $\left(-\frac{1}{3}, \frac{128}{27}\right)$

d.Find all relative extrema by first derivative test where applicable.

Local minimum at $x = 1$ at point $(1, 0)$.

Local maximum at $x = -\frac{5}{3}$ at point $\left(-\frac{5}{3}, \frac{256}{27}\right)$.

e.Determine the open intervals on which the function is concave upward or concave downward.

Concave upward at $\left(-\frac{1}{3}, \infty\right)$.

Concave downward at $\left(-\infty, -\frac{1}{3}\right)$.

11. For the given function $f(x) = 4 - x - 3x^4$
    a. Find the critical numbers.

$$c = -\frac{1}{\sqrt[3]{12}}.$$

b. Find the open intervals on which the function is increasing or decreasing.

Increasing interval $\left(-\infty, -\frac{1}{\sqrt[3]{12}}\right)$.

Decreasing interval $\left(-\frac{1}{\sqrt[3]{12}}, \infty\right)$.

c. Find the inflection points.

Inflection point $(0,4)$.

d. Find all relative extrema by first derivative test where applicable.

Absolute maximum at $x = -\frac{1}{\sqrt[3]{12}}$ at point $\left(-\frac{1}{\sqrt[3]{12}}, \frac{\left(3+16\sqrt[3]{12}\right)}{4\sqrt[3]{12}}\right)$.

12. Find the rate of change $\frac{dS}{dt}$ of the function $S = x^3 + 4y$ when $x = 5, y = -6, \frac{dx}{dt} = -1$, and $\frac{dy}{dt} = 2$.

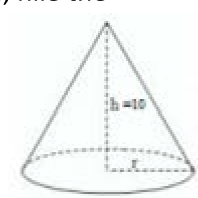
Solution:

$$\frac{ds}{dt} = 3x^2\frac{dx}{dt} + 4\frac{dy}{dt}$$
$$= 3\;5^2\;(-1) + 4\;2 = \boxed{-67}$$

13. A rectangular box has base width twice the size of its base length. The volume of this box is 72 cubic units. Find the dimensions that will minimize surface area.

Length: 3, width: 6, height: 4.

14. The radius r of a right circular cone is increasing at a rate of 6 inches per minute, hile the height h of the cone remains constant at 10 inches. Find the rate of change of the volume V with respect to the time t, when $r = 12$ inches. The volume of a right circular cone is $V = \frac{\pi}{3}r^2h$.



$$\frac{dV}{dt} = \frac{\pi}{3}\,2r\frac{dt}{dt}h = \frac{\pi}{3}(2)(12)(6)(10) = 480\pi.$$

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15. Find all horizontal and vertical asymptotes.

a. $f(x) = \frac{x}{x^2-4}$

Horizontal asymptotes:

$y = 0$.

Vertical asymptotes:

$x = -2, x = 2$

b. $f(x) = \frac{x^2}{x-3}$

Horizontal asymptotes:

DNE

Vertical asymptotes:

$x = 3$

16. Find an indefinite integral.

a. $\int (1 + 6x)^4 \, dx$

$= \frac{1}{30} (1 + 6x)^5 + c$

b. $\int x(x^2 - 9)^3 \, dx$

$= \frac{1}{8}(x^2 - 9)^4 + c$

c. $\int t^3\sqrt{2t^4 + 4} \, dt$

$= -\frac{1}{4}(2t^4 + 4)^{-\frac{1}{2}} + c$

d. $\int \frac{x^2}{(4x^3-9)^3} \, dx$

$= -\frac{1}{24(4x^3-9)^2} + c$

17. Evaluate the definite integral

a. $\int_0^4 \frac{1}{\sqrt{2x+1}} \, dx$

$= \sqrt{2x + 1} \,|_1^4$

$= 2$

b. $\int_1^2 2x^2\sqrt{x^3 + 1} \, dx$

$= \frac{4}{9}(x^3 + 1)^{\frac{3}{2}} \,|_1^2$

$= 12 - \frac{8}{9}\sqrt{2}$

18. Evaluate the integral using the properties of even and odd functions.

a. $\int_{-2}^2 x^2(x^2 + 1) \, dx$

$= \frac{272}{15}$.

b. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x \cos x \, dx$

$= 0$.

19. Differentiate the function.

a. $\int_0^x t^2 \sin t \, dt$

$F(x) = \int_0^x t^2 \sin t \, dt$

$F'(x) = x^2 \sin x$

b. $\int_2^{x^2} t + 5 \, dt$

$F(x) = \int_2^{x^2} t + 5 \, dt$

$F'(x) = x^2 + 5$

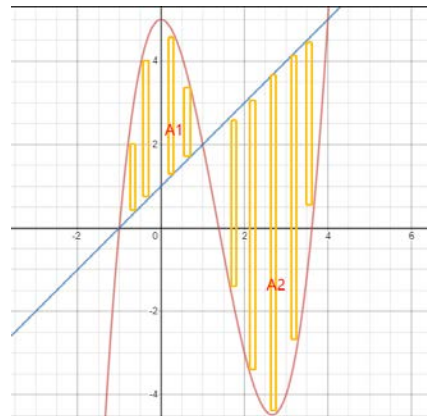
20. Find the area of region $A1$ and area of region $A2$, between the curves $y = x + 1$ and $y = x^3 - 4x^2 + 5$

Find the intersection of two curves set $x + 1 = x^3 - 4x^2 + 5$

Obtain $x = -1, 1, 4.$

$$A1 = \int_{-1}^{1} (x^3 - 4x^2 + 5) - (x + 1)\,dx = \frac{16}{3}$$

$$A2 = \int_{1}^{4} (x + 1) - (x^3 - 4x^2 + 5)\,dx = \frac{63}{4}$$



21. Find the area bounded by the graphs of the equations.
$f = \cos^2 x$, $g = \sin x \cos x$, $x = -\frac{\pi}{2}$, and $x = \frac{\pi}{4}$.

Area $= \int_{-\frac{\pi}{2}}^{\frac{\pi}{4}} \cos^2 x - \sin x \cos x \;dx = \frac{3\pi+4}{8}$