$$\frac{d}{dx}(\ln|u|) = \frac{u'}{u}$$

$$\frac{d}{dx}(a^u) = (\ln a) \cdot a^u \cdot u'$$

$$\frac{d}{dx}\log_a u = \frac{u'}{u \cdot (\ln a)}$$

Basic Integration Rules:

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C, n \neq -1$$

$$\int \sin u \, du = -\cos u + C$$

$$\int \cos u \, du = \sin u + C$$

$$\int \tan u \, du = -\ln|\cos u| + C$$

$$\int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \csc u \, du = -\ln|\csc u + \cot u| + C$$

$$\int e^u dx = e^u + C$$

$$\int \frac{u'}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C$$

$$\int \frac{u'}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{|u|}{a} + C$$

$$\int \frac{u'}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

Integration by Substitution:

$$\int_a^b f\big(g(x)\big) dx = \int_{g(a)}^{g(b)} f(u) du$$
 , where $u=g(x)$ and $du=g'(x) dx$

L'Hopital's Rule: When taking a limit, if you get an indeterminate form i.e. $\frac{\pm \infty}{\pm \infty}$, $\frac{0}{0}$, then take the derivative of the top and bottom and evaluate the limit again.

Integration by Parts:

$$\int u dv = uv - \int v du$$
, where $v = \int dv$

Trig Substitution:

If the integral contains the following root, use the given substitution and formula to convert into an integral involving trig functions.

$$\sqrt{a^2 - u^2} \Rightarrow u = a \sin \theta$$
$$\sqrt{u^2 - a^2} \Rightarrow u = a \sec \theta$$
$$\sqrt{a^2 + u^2} \Rightarrow u = a \tan \theta$$

Trig References:

$$\sin^2 x + \cos^2 x = 1$$

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

Improper Integral:

Convergent: Limit exists as a finite number. Divergent: Limit exists as either $\pm \infty.$

Arc Length:

$$s = \int_{a}^{b} \sqrt{1 + [f'(x)]^2} \, dx$$

Product and Quotients of Trig Functions:

- a. $\int \sin^m x \cos^n x \, dx$
 - 1. n is odd: save one $\cos x$ and use $\cos^2 x = 1 \sin^2 x$ to express the remaining factors in terms of sine.
 - 2. m is odd: save one $\sin x$ and use $\sin^2 x = 1 \cos^2 x$ to express the remaining factors in terms of cosine.
 - 3. If both m and n are even, use the half angle identities.
- b. $\int \tan^m x \sec^n x \, dx$
 - 1. m is odd: save one $\sec x \tan x$ and use $\tan^2 x = \sec^2 x 1$ to express the remaining factors in terms of secant.
 - 2. n is even: save one $\sec^2 x$ and use $\sec^2 x = 1 + \tan^2 x$ to express the remaining factors in terms of tangent.

Integral of a Rational Function: $\int \frac{p(x)}{q(x)} dx$

If the degree of $p(x) \ge$ degree of q(x), use long division.

If the degree of p(x) < degree of q(x), use partial fractions.

$\frac{d}{dx}(\ln|u|) = \frac{u'}{u}$ $\frac{d}{dx}(a^u) = (\ln a) \cdot a^u \cdot u'$ $\frac{d}{dx}\log_a u = \frac{u'}{u \cdot (\ln a)}$

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Improper Integral:

Convergent: Limit exists as a finite number. Divergent: Limit exists as either $\pm \infty$.

Arc Length:

$$s = \int_{a}^{b} \sqrt{1 + [f'(x)]^2} \, dx$$

Product and Quotients of Trig Functions:

- c. $\int \sin^m x \cos^n x \, dx$
 - 4. n is odd: save one $\cos x$ and use $\cos^2 x = 1 \sin^2 x$ to express the remaining factors in terms of sine.
 - 5. m is odd: save one $\sin x$ and use $\sin^2 x = 1 \cos^2 x$ to express the remaining factors in terms of cosine.
 - 6. If both m and n are even, use the half angle identities.
- d. $\int \tan^m x \sec^n x \, dx$
 - 3. m is odd: save one $\sec x \tan x$ and use $\tan^2 x = \sec^2 x 1$ to express the remaining factors in terms of secant.
 - 4. n is even: save one $\sec^2 x$ and use $\sec^2 x = 1 + \tan^2 x$ to express the remaining factors in terms of tangent.

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