

LIMITS	DERIVATIVES	INTEGRALS
<p>Informal Definition of a Limit: The behavior of $f(x)$ as x approaches a value c, from left and right.</p> $\lim_{x \rightarrow c} f(x) = L$ <p>Formal Definition of a Limit: Let f be defined on an open interval containing c (except possibly at c), and let L be a real number. The statement</p> $\lim_{x \rightarrow c} f(x) = L$ <p>means that for every $\varepsilon > 0$, there exists a $\delta > 0$, such that if $0 < x - c < \delta$, then $f(x) - L < \varepsilon$.</p> <p>Properties of Limits:</p> $\lim_{x \rightarrow a} [c \cdot f(x)] = c \cdot \lim_{x \rightarrow a} f(x)$ $\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$ $\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$ $\lim_{x \rightarrow a} \left(\frac{f(x)}{g(x)} \right) = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}, \lim_{x \rightarrow a} g(x) \neq 0$ $\lim_{x \rightarrow a} [f(x)]^n = [\lim_{x \rightarrow a} f(x)]^n$ <p>Limit Evaluations at Infinity for Rational Functions:</p> $R(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials. <ol style="list-style-type: none"> If degree of $p(x) <$ degree of $q(x)$, then $\lim_{x \rightarrow \pm\infty} \frac{p(x)}{q(x)} = 0$. If degree of $p(x) =$ degree of $q(x)$, then $\lim_{x \rightarrow \pm\infty} \frac{p(x)}{q(x)} =$ ratio of the leading coefficient. If degree of $p(x) >$ degree of $q(x)$, then $\lim_{x \rightarrow \pm\infty} \frac{p(x)}{q(x)} = \infty$ or $-\infty$, depending on the sign of the leading coefficient. <p>Continuity at a point: A function $f(x)$ is continuous at a point if $\lim_{x \rightarrow c} f(x)$ exists, $f(c)$ is defined and $\lim_{x \rightarrow c} f(x) = f(c)$.</p> <p>Derivative definition:</p> $\frac{d}{dx} f(x) = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \text{slope of the tangent line}$	<p>Basic Properties:</p> $(c \cdot f(x))' = c \cdot f'(x)$ $(f(x) \pm g(x))' = f'(x) \pm g'(x)$ $\frac{d}{dx} (\text{constant}) = 0$ <p>Power Rule:</p> $\frac{d}{dx} (x^n) = n \cdot x^{n-1}$ <p>Product Rule:</p> $(f(x) \cdot g(x))' = f'(x)g(x) + f(x)g'(x)$ <p>Quotient Rule:</p> $\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$ <p>Chain Rule:</p> $\frac{d}{dx} (f(g(x))) = f'(g(x)) \cdot g'(x)$ <p>Common Derivatives:</p> $\frac{d}{dx} (\sin x) = \cos x$ $\frac{d}{dx} (\cos x) = -\sin x$ $\frac{d}{dx} (\tan x) = \sec^2 x$ $\frac{d}{dx} (\sec x) = \sec x \tan x$ $\frac{d}{dx} (\csc x) = -\csc x \cot x$ $\frac{d}{dx} (\cot x) = -\csc^2 x$ $\frac{d}{dx} (e^x) = e^x$ $\frac{d}{dx} (\ln x) = \frac{1}{x}$	<p>Definite Integral Definition:</p> $\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x$, where $\Delta x = \frac{b-a}{n}$ and $x_k = a + k\Delta x$ <p>Fundamental Theorem of Calculus:</p> $\int_a^b f(x) dx = F(b) - F(a)$ where f is continuous on $[a, b]$ and $F' = f$. <p>Integration Properties:</p> $\int_a^b c f(x) dx = c \int_a^b f(x) dx$ $\int_a^b f(x) \pm g(x) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$ $\int_a^a f(x) dx = 0$ and $\int_a^b f(x) dx = -\int_b^a f(x) dx$ $\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$ <p>Power rule for Integration:</p> $\int x^n dx = \frac{1}{n+1} x^{n+1} + C, n \neq -1$ <p>Common Integrals:</p> $\int \cos x dx = \sin x + C$ $\int \sin x dx = -\cos x + C$ $\int \sec^2 x dx = \tan x + C$ $\int \sec x dx = \sec x \tan x + C$ $\int \csc x \cot x dx = -\csc x + C$ $\int \csc^2 x dx = -\cot x + C$ $\int e^x dx = e^x + C$ $\int \frac{1}{x} dx = \ln x + C$ <p>Integration by Substitution:</p> $\int_a^b f(g(x)) dx = \int_{g(a)}^{g(b)} f(u) du$, where $u = g(x)$ and $du = g'(x) dx$

LIMITS	DERIVATIVES	INTEGRALS
<p>Informal Definition of a Limit: The behavior of $f(x)$ as x approaches a value c, from left and right.</p> $\lim_{x \rightarrow c} f(x) = L$ <p>Formal Definition of a Limit: Let f be defined on an open interval containing c (except possibly at c), and let L be a real number. The statement</p> $\lim_{x \rightarrow c} f(x) = L$ <p>means that for every $\varepsilon > 0$, there exists a $\delta > 0$, such that if $0 < x - c < \delta$, then $f(x) - L < \varepsilon$.</p> <p>Properties of Limits:</p> $\lim_{x \rightarrow a} [c \cdot f(x)] = c \cdot \lim_{x \rightarrow a} f(x)$ $\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$ $\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$ $\lim_{x \rightarrow a} \left(\frac{f(x)}{g(x)} \right) = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}, \lim_{x \rightarrow a} g(x) \neq 0$ $\lim_{x \rightarrow a} [f(x)]^n = [\lim_{x \rightarrow a} f(x)]^n$ <p>Limit Evaluations at Infinity for Rational Functions:</p> $R(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials. <ol style="list-style-type: none"> If degree of $p(x) <$ degree of $q(x)$, then $\lim_{x \rightarrow \pm\infty} \frac{p(x)}{q(x)} = 0$. If degree of $p(x) =$ degree of $q(x)$, then $\lim_{x \rightarrow \pm\infty} \frac{p(x)}{q(x)} =$ ratio of the leading coefficient. If degree of $p(x) >$ degree of $q(x)$, then $\lim_{x \rightarrow \pm\infty} \frac{p(x)}{q(x)} = \infty$ or $-\infty$, depending on the sign of the leading coefficient. <p>Continuity at a point: A function $f(x)$ is continuous at a point if $\lim_{x \rightarrow c} f(x)$ exists, $f(c)$ is defined and $\lim_{x \rightarrow c} f(x) = f(c)$.</p> <p>Derivative definition:</p> $\frac{d}{dx} f(x) = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \text{slope of the tangent line}$	<p>Basic Properties:</p> $(c \cdot f(x))' = c \cdot f'(x)$ $(f(x) \pm g(x))' = f'(x) \pm g'(x)$ $\frac{d}{dx} (\text{constant}) = 0$ <p>Power Rule:</p> $\frac{d}{dx} (x^n) = n \cdot x^{n-1}$ <p>Product Rule:</p> $(f(x) \cdot g(x))' = f'(x)g(x) + f(x)g'(x)$ <p>Quotient Rule:</p> $\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$ <p>Chain Rule:</p> $\frac{d}{dx} (f(g(x))) = f'(g(x)) \cdot g'(x)$ <p>Common Derivatives:</p> $\frac{d}{dx} (\sin x) = \cos x$ $\frac{d}{dx} (\cos x) = -\sin x$ $\frac{d}{dx} (\tan x) = \sec^2 x$ $\frac{d}{dx} (\sec x) = \sec x \tan x$ $\frac{d}{dx} (\csc x) = -\csc x \cot x$ $\frac{d}{dx} (\cot x) = -\csc^2 x$ $\frac{d}{dx} (e^x) = e^x$ $\frac{d}{dx} (\ln x) = \frac{1}{x}$	<p>Definite Integral Definition:</p> $\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x$, where $\Delta x = \frac{b-a}{n}$ and $x_k = a + k\Delta x$ <p>Fundamental Theorem of Calculus:</p> $\int_a^b f(x) dx = F(b) - F(a)$ where f is continuous on $[a, b]$ and $F' = f$. <p>Integration Properties:</p> $\int_a^b c f(x) dx = c \int_a^b f(x) dx$ $\int_a^b f(x) \pm g(x) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$ $\int_a^a f(x) dx = 0$ and $\int_a^b f(x) dx = -\int_b^a f(x) dx$ $\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$ <p>Power rule for Integration:</p> $\int x^n dx = \frac{1}{n+1} x^{n+1} + C, n \neq -1$ <p>Common Integrals:</p> $\int \cos x dx = \sin x + C$ $\int \sin x dx = -\cos x + C$ $\int \sec^2 x dx = \tan x + C$ $\int \sec x dx = \sec x \tan x + C$ $\int \csc x \cot x dx = -\csc x + C$ $\int \csc^2 x dx = -\cot x + C$ $\int e^x dx = e^x + C$ $\int \frac{1}{x} dx = \ln x + C$ <p>Integration by Substitution:</p> $\int_a^b f(g(x)) dx = \int_{g(a)}^{g(b)} f(u) du$, where $u = g(x)$ and $du = g'(x) dx$