

SAMPLE PLACEMENT TEST TO BYPASS MATH 1404
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1. Give the equation of the line passing through the points (2, 2) and (6, -3).
2. Find the inverse function of $f(x) = \frac{2x-5}{x-3}$ if it exists.
3. Find all of the roots of $3x^3 + 4x^2 - 35x - 12 = 0$.
4. Solve for x : $\log(28+x) = \log(2-x) + \log(2-x)$.
5. Solve for x : $7^{3x+1} = 100$.
6. Solve the following trigonometric equation for all values of x such that $0 \leq x < 2\pi$.
 $\cos x + 2 \sin x \cos x = 0$
7. Write the first four terms of the binomial expansion of $(a-2b)^{12}$.
8. Identify the conic given by the following equation and give its center.
 $16x^2 - 9y^2 - 128x - 18y + 103 = 0$
9. Find the points of intersection of the following polar curves.
$$\begin{cases} r = 1 + \sin \theta \\ r = 1 + \cos \theta \end{cases}$$
10. Given $f(x) = x^2$ and $g(x) = 3x + 1$, find $f(g(x))$.
11. If $f(x) = \frac{2x+3}{x-1}$, find $f(-2)$.
12. Give the equation of the hyperbola with vertices at (4, 4) and (4, 8), and foci at (4, 3) and (4, 9).
13. Give the value of $\log_3 27$.
14. Find the sum of the first 10 terms of the arithmetic sequence in which the first term is 25 and the common difference is -2.
15. Simplify the following expression by using the fundamental identities: $\frac{(1 - \cos^2 \theta)}{\sin \theta}$.
16. Find all roots of $x^4 - 6x^3 + 11x^2 - 2x - 10 = 0$, given that $2 - i$ is a root of f .
17. Give the value of $\text{Arctan}(-1)$.
18. Find the sum of the first five terms of the geometric sequence with first term 3 and common ratio 2.
19. Give the equation of an ellipse with vertices (-6, 3) and (4, 3), and foci at (-3, 3) and (1, 3).
20. Find the sum of an infinite geometric sequence with first term 2 and common ratio $\frac{1}{2}$.
21. Use DeMoivre's Theorem to find $(-1 + \sqrt{3}i)^{12}$.
22. For the following parametric equations, write the corresponding rectangular equation by eliminating the parameter: $x = \ln 2t$, $y = t^2$.
23. Use mathematical induction to prove that the following statement is true for every natural number n .
 $3 + 9 + 15 + \dots + (6n - 3) = 3n^2$
24. Find the fourth roots of $-2\sqrt{3} + 2i$. Leave the answers in trigonometric form.
25. Find the following quotient. Give the answer in trigonometric form, and also in rectangular form.
$$\frac{\sqrt{36}(\cos 365^\circ + i \sin 365^\circ)}{\sqrt{2}(\cos 50^\circ + i \sin 50^\circ)}$$

In problems 26 — 31, sketch the graph of the given function or equation.

- | | |
|--------------------------------|--|
| 26. $f(x) = \frac{1}{x-2} + 2$ | 27. $r = 3 - \sin \theta$ |
| 28. $y = e^x + 2$ | 29. $y = \begin{cases} 3 & \text{if } x \leq 0 \\ -x^2 - 2 & \text{if } x > 0 \end{cases}$ |
| 30. $y = x^3 + 2x^2 - 9x - 18$ | 31. $y = \log_{1/2}(x+3)$ |

1. $5x + 4y - 18 = 0$
3. $x = 3, x = -\frac{1}{3},$ and $x = -4$
5. $x = \frac{2}{3 \log 7} - \frac{1}{3}$
7. $a^{12} - 24a^{11}b + 264a^{10}b^2 - 1760a^9b^3 + \dots$

9. $(\frac{2 + \sqrt{2}}{2}, \frac{\pi}{4}), (\frac{2 - \sqrt{2}}{2}, \frac{5\pi}{4})$.

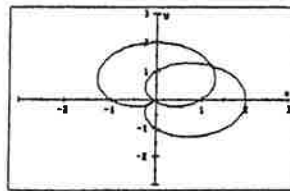
By sketching the curves,
we see that $(0, 0)$ is a solution also.

2. $f^{-1}(x) = \frac{3x-5}{x-2}$

4. $x = -3$

6. $x = : \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6},$ or $\frac{11\pi}{6}$

8. A hyperbola with center $(4, -1)$.



10. $f(g(x)) = 9x^2 + 6x + 1$

12. $\frac{(y-6)^2}{4} - \frac{(x-4)^2}{5} = 1$

14. 160

16. $2 \pm i, 1 \pm \sqrt{3}$

18. 93

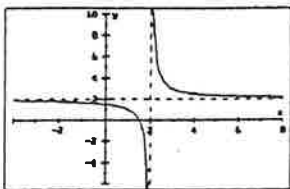
20. 4

22. $y = \frac{e^{2x}}{4}$

24. $\sqrt[4]{4}(\cos \frac{5\pi}{24} + i \sin \frac{5\pi}{24}), \sqrt[4]{4}(\cos \frac{17\pi}{24} + i \sin \frac{17\pi}{24}), \sqrt[4]{4}(\cos \frac{29\pi}{24} + i \sin \frac{29\pi}{24}), \sqrt[4]{4}(\cos \frac{41\pi}{24} + i \sin \frac{41\pi}{24})$

25. $3\sqrt{2}(\cos 315^\circ + i \sin 315^\circ), 3 - 3i$

26.

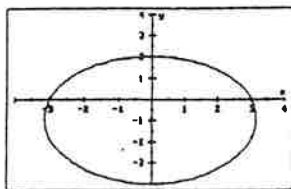


$x = 2$ is a vertical asymptote.

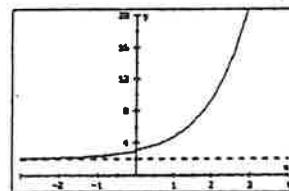
$y = 2$ is a horizontal asymptote.

The intercepts are $(0, \frac{3}{2})$ and $(\frac{3}{2}, 0)$.

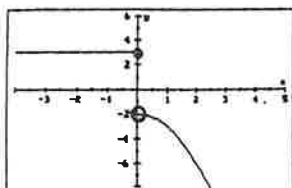
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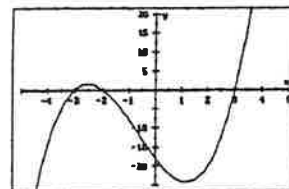
28.



29.



30.



31.

