SAMPLE PLACEMENT TEST TO BYPASS MATH 1404
Department of Applied Mathematical Sciences, University of Houston-Downtown

1. Give the equation of the line passing through the points $(2, 2)$ and $(6, -3)$.

2. Find the inverse function of $f(x) = \dfrac{2x-5}{x-3}$ if it exists.

3. Find all of the roots of $3x^3 + 4x^2 - 35x - 12 = 0$.

4. Solve for x: $\log(28 + x) = \log(2 - x) + \log(2 - x)$.

5. Solve for x: $7^{3x+1} = 100$.

6. Solve the following trigonometric equation for all values of x such that $0 \le x < 2\pi$.

   $\cos x + 2 \sin x \cos x = 0$

7. Write the first four terms of the binomial expansion of $(a - 2b)^{12}$.

8. Identify the conic given by the following equation and give its center.

   $16x^2 - 9y^2 - 128x - 18y + 103 = 0$

9. Find the points of intersection of the following polar curves.
   $$\begin{cases} r = 1 + \sin\theta \\ r = 1 + \cos\theta \end{cases}$$

10. Given $f(x) = x^2$ and $g(x) = 3x + 1$, find $f(g(x))$.

11. If $f(x) = \dfrac{2x+3}{x-1}$, find $f(-2)$.

12. Give the equation of the hyperbola with vertices at $(4, 4)$ and $(4, 8)$, and foci at $(4, 3)$ and $(4, 9)$.

13. Give the value of $\log_3 27$.

14. Find the sum of the first 10 terms of the arithmetic sequence in which the first term is 25 and the common difference is -2.

15. Simplify the following expression by using the fundamental identities: $\dfrac{(1 - \cos^2\theta)}{\sin\theta}$.

16. Find all roots of $x^4 - 6x^3 + 11x^2 - 2x - 10 = 0$, given that $2 - i$ is a root of f.

17. Give the value of $\operatorname{Arctan}(-1)$.

18. Find the sum of the first five terms of the geometric sequence with first term 3 and common ratio 2.

19. Give the equation of an ellipse with vertices $(-6, 3)$ and $(4, 3)$, and foci at $(-3, 3)$ and $(1, 3)$.

20. Find the sum of an infinite geometric sequence with first term 2 and common ratio $\frac{1}{2}$.

21. Use DeMoivre's Theorem to find $(-1 + \sqrt{3}\,i)^{12}$.

22. For the following parametric equations, write the corresponding rectangular equation by eliminating the parameter: $x = \ln 2t$, $y = t^2$.

23. Use mathematical induction to prove that the following statement is true for every natural number n.

    $3 + 9 + 15 + \cdots + (6n - 3) = 3n^2$

24. Find the fourth roots of $-2\sqrt{3} + 2i$. Leave the answers in trigonometric form.

25. Find the following quotient. Give the answer in trigonometric form, and also in rectangular form.

    $\dfrac{\sqrt{36}(\cos 365° + i \sin 365°)}{\sqrt{2}(\cos 50° + i \sin 50°)}$

In problems 26 — 31, sketch the graph of the given function or equation.

26. $f(x) = \dfrac{1}{x-2} + 2$

27. $r = 3 - \sin\theta$

28. $y = e^x + 2$

29. $y = \begin{cases} 3 & \text{if } x \le 0 \\ -x^2 - 2 & \text{if } x > 0 \end{cases}$

30. $y = x^3 + 2x^2 - 9x - 18$

31. $y = \log_{1/2}(x + 3)$

1. $5x + 4y - 18 = 0$

2. $f^{-1}(x) = \dfrac{3x - 5}{x - 2}$

3. $x = 3$, $x = -\dfrac{1}{3}$, and $x = -4$

4. $x = -3$

5. $x = \dfrac{2}{3\log 7} - \dfrac{1}{3}$

6. $x = : \dfrac{\pi}{2},\ \dfrac{3\pi}{2},\ \dfrac{7\pi}{6},$ or $\dfrac{11\pi}{6}$

7. $a^{12} - 24a^{11}b + 264a^{10}b^2 - 1760a^9b^3 + \cdots$

8. A hyperbola with center $(4,\ -1)$.

9. $\left(\dfrac{2 + \sqrt{2}}{2}, \dfrac{\pi}{4}\right), \left(\dfrac{2 - \sqrt{2}}{2}, \dfrac{5\pi}{4}\right)$.

By sketching the curves,

we see that $(0,\ 0)$ is a solution also.

10. $f(g(x)) = 9x^2 + 6x + 1$

11. $f(-2) = \dfrac{1}{3}$

12. $\dfrac{(y - 6)^2}{4} - \dfrac{(x - 4)^2}{5} = 1$

13. 3

14. 160

15. $\sin\theta$

16. $2 \pm i,\ 1 \pm \sqrt{3}$

17. $-\dfrac{\pi}{4}$

18. 93

19. $\dfrac{(x + 1)^2}{25} + \dfrac{(y - 3)^2}{21} = 1$
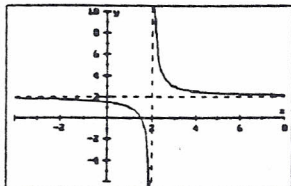
20. 4

21. 4096
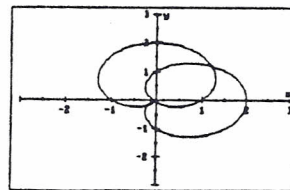
22. $y = \dfrac{e^{2x}}{4}$

24. $\sqrt[4]{4}\left(\cos\dfrac{5\pi}{24} + i\sin\dfrac{5\pi}{24}\right), \sqrt[4]{4}\left(\cos\dfrac{17\pi}{24} + i\sin\dfrac{17\pi}{24}\right), \sqrt[4]{4}\left(\cos\dfrac{29\pi}{24} + i\sin\dfrac{29\pi}{24}\right), \sqrt[4]{4}\left(\cos\dfrac{41\pi}{24} + i\sin\dfrac{41\pi}{24}\right)$
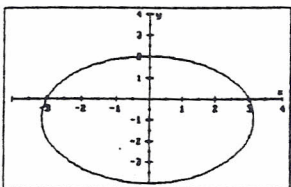
25. $3\sqrt{2}(\cos 315° + i\sin 315°),\ 3 - 3i$

26.

$x = 2$ is a vertical asymptote.

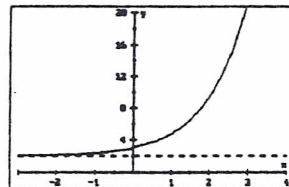$y = 2$ is a horizontal asymptote.

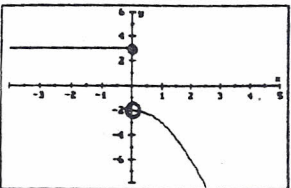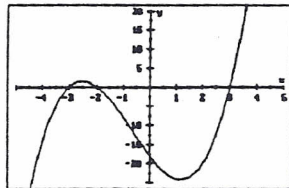The intercepts are $\left(0, \dfrac{3}{2}\right)$ and $\left(\dfrac{3}{2}, 0\right)$.

27.

28.

29.

30.

31.