Minimum-Multiplicity Routing Problems of Multimedia Communications

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ABSTRACT

In this work, we investigate the routing problems of minimizing the number of distinct transmission mediums or carrier domains for multimedia communications. We consider the problems of two-party connection as well as three-or-more-party connections. By minimizing the number of distinct transmission mediums or carrier domains on the connections, rather than minimizing bandwidth, length or hop counts, the network carriers may achieve higher cost savings on connection provisioning and management. These problems are proven to be NP-hard. Heuristic algorithms are proposed for larger instances of the problems and evaluated through simulations.

1. INTRODUCTION

A carrier's network normally spans across large geographical areas and connects millions of customers including both businesses and residential users. With the rapid advances of net-enabled multimedia applications, more and more network traffic are generated by multimedia communications such as video conferencing, internet telephony, online gaming and video/audio-on-demand [1][2][3], which has prompted intense research activities in those areas. Many researches concentrate on the content streaming issues such as authentication, encoding/decoding and loss compensation etc.[4][5][6], while others concentrate on the network infrastructure issues such as GMPLS and broadband switching, optical wireless [7][8][9][10].

For the network carriers, multimedia communication services are becoming a major source of income growth. To satisfy the quality-of-service (QoS) requirements, the carriers often utilize services such as IntServ to provision and manage communication paths [11] between two or more end nodes of their networks. Hence, it is highly desirable for the carriers to support as many connections as possible with minimal cost. One common approach to minimize cost is to allocate the least amount of network resources such as bandwidth, path length or hop count for the connection paths while satisfying the QoS requirement. This philosophy leads to the deployment of minimum-cost-path (i.e. shortest-path) based routing protocols such as RIP and OSPF for communication between two endnodes [12][13][14][15]. If three or more parties are involved in a communication session such as in a video conferencing or online gaming, it is then desirable to find a minimum-cost tree that connects all the end nodes for each party. This is categorized as a Steiner Tree problem. This problem has been proven NP-hard, for which approximation algorithms have been developed [16].

In reality, many factors hinder the effectiveness of the

minimum-cost-based approaches. One of the major obstacles is the heterogeneous nature of carrier networks. For example, within a carrier network, there often co-exist both legacy transmission mediums such as SONET, X.25 and Frame Relay [17][18][19], as well as newer technologies such as WDM [20], GMPLS and broadband wireless, etc. If the shortest-path based approach finds a path that has the minimum bandwidth, length or hop count but traverses many different types of transmission mediums, it may actually cost more for a carrier to provision and manage than for a different path that traverses fewer types of transmission mediums even if the path is suboptimal in bandwidth, length or hop count, given that the QoS requirements are still satisfied. Similarly, a cross-nation or cross-continent connection may require the collaboration of multiple carriers. A communication path is often less costly to provision and manage if it traverses fewer carrier domains.

In this paper we investigate four routing problems of multimedia communications in carrier networks with the objective of minimizing the number of distinct transmission mediums or carrier domains. Theses problems are categorized as minimum-multiplicity routing problems. We consider the problems of two-party connection as well as three-or-moreparty connections. The rest of the paper is organized as follows. Section 2 discuss the four problems and prove their NPhardness. Section 3 presents heuristics and evaluates their performance through computer simulations. Section 4 concludes the paper.

2. MINIMUM-MULTIPLICITY ROUTING PROBLEMS

We investigate four minimum-multiplicity routing problems. Throughout the discussion, we assume all connection requests and network links are bidirectional.

2.1. Minimum-Multiplicity Single Connection Routing

In this problem, we try to find a path connecting two end-points of a network while minimizing the number of distinct transmission mediums or carrier domains. This problem is formally defined as follows. Given network G = (N, L), where N is the set of nodes and L is the set of links, and given the set of labels $C = \{c_1, c_2, c_3, ..., c_K\}$ where each label represents a distinct transmission medium or carrier domain and K is the maximum number of labels in G, and given the label $c_l \in C$ for every link $l \in L$, find one path from source node s to destination node d such that it uses the minimum number of distinct labels.

We need to reduce a known NP-hard problem to this problem. The known NP-hard problem in this case is the Minimum Set Covering Problem [21]. This problem is stated as follows. Given a finite set $S = \{a_1, a_2, a_3, ..., a_n\}$, and a collection $C = \{C_1, C_2, ..., C_m\}$ such that each element in *C* contains a subset of

S, is there a minimum subset, $C' \subseteq C$ such that every member of S belong to at least one member of C'?

We construct a graph G for an arbitrary instance of the Minimum Set Covering Problem, such that the graph contains one path from s to d with the minimum number of labels, if and only if C contains a minimum set cover C'. Following are the steps for the graph construction:

Step 1. For every element a_i in S, create a network node a_i . Step 2. For every subset C_j to which a_i belongs, create a network link a_{i-1} a_i of label c_j . For element a_1 , the link is sa_1 . There is also a single link between a_n and d with label c_0 .

An example is given in Fig 1. In this example, we construct graph G for a Minimum Set Covering problem $S = \{a_1, a_2, a_3, a_4\}, C = \{C_1, C_2, C_3, C_4, C_5\}, C_1 = \{a_1, a_2\}, C_2 = \{a_2, a_3\}, C_3 = \{a_1, a_3\}, C_4 = \{a_3, a_4\}, C_5 = \{a_1, a_4\}.$



Fig. 1. Reduction of the Minimum Set Covering Problem

It is obvious that if there is a path from s to d with minimum number of distinct labels, then the labels on that path are mapped directly to a minimum set covering all the elements in S. Conversely, if there is a minimum set covering all the elements, then a path with minimum number of different labels can be derived by going through every node and selecting the link with the label representing the set that covers the corresponding element. Hence, the minimum-multiplicity single connection routing problem is NP-hard.

2.2. Minimum-Multiplicity Tree Routing

As described in Section 1, when three or more participants exist in a multimedia communication session such as video conferencing or online gaming, the minimum-cost-based Steiner tree approach may generate connections that traverse many different types of transmission mediums or carrier domains which are costly to provision and manage. Rather, we prefer a tree connecting those end-points while minimizing the number of distinct transmission mediums or carrier domains traversed by the tree.

This problem is easily reducible to a known NP-hard minimum-labeling spanning tree problem [22] by making all network nodes the participant end nodes of a communication session. Hence, this problem is NP-hard.

2.3. Minimum-Multiplicity Ring-Connection Routing

If the connections between the communication participants form a tree, the failure of a single tree node or tree link may segment the connections [23][24]. To achieve better fault tolerance, we can resort to ring topology which survives single link or node failure. Now the problem becomes finding a ring connecting all participants while minimizing the number of distinct transmission mediums or carrier domains traversed by the ring. This problem is reducible to the Hamiltonian-cycle problem, which is a known NP-hard problem by the follow

steps:

Step 1. For the network graph G of a Hamiltonian-cycle problem, construct an identical network graph G'

Step 2. Assign a unique transmission medium or carrier domain to every link and node in G'

Hence, a minimum-multiplicity ring connecting all nodes in G' is equivalent to Hamiltonian-cycle in G and this problem is proven NP-hard.

2.4. Minimum-Multiplicity Mesh-Connection Routing

A ring topology provides fault tolerance for single link or node failure. For multimedia connections that require higher level of fault-tolerance, a carrier may offer the option of full-mesh connections in which every participant has a direct connection to all other participants. Full-mesh minimum-cost connections can be easily obtained by finding the shortest path between every pair of participants. On the other hand, finding full-mesh connections with minimum total number of distinct transmission mediums or carrier domains is NP-hard since it contains the minimum-multiplicity single connection problem as a special case.

3. HEURISTICS AND SIMULATIONS

We give two heuristics to solve the minimum-multiplicity single connection routing problem. The first heuristic is called Single-Path Reduction Algorithm (SPRA). In this algorithm, we first run Dijkstra's algorithm or the Bellman-Ford algorithm to find the shortest path from the source s to the destination d. We then try to eliminate some of the labels while still being able to find a path from s to d. The details are as follows.

Step 1.Run a shortest path algorithm and find a path p. Assume that the collection of all the labels on p is set $C_p = \{c_1, c_2, ..., c_k\}$.

Step 2. Go through every label in C_p . Select the label such that, after the links of that label are removed from the network, we run the shortest path algorithm and obtain a shortest path with the minimum number of labels which is also less than $|C_p|$. Remove the links of the selected label.

Step 3. Repeat Step 1 and 2 until the number of labels on the shortest path cannot be further reduced.

The running time is $O(m^2 n \log n)$ where *n* is the number of nodes and *m* is the total number of labels in the network.

The next heuristic is called the Single-Path Optimization Algorithm (SPOA). In this algorithm, we go through all the labels and try to use only a subset of them on paths from s to d. The details are as follows.

Step 1. Run a shortest path algorithm and find a path p. Assume the number of labels on p is $|C_p|$.

Step 2. Set the link cost to zero on the links of one label, and find the shortest path. Repeat for all the labels in the network and select the one that results in a path with the minimum number of labels which is also less than $|C_p|$. Keep the costs to zero on the links of the selected label.

Step 3. Repeat Step 1 and 2 until the number of labels on the shortest paths cannot be further reduced.

The running time is $O(m^2 n \log n)$ where *n* is the number of nodes and *m* is the total number of labels in the network.



Fig 2. Average number of distinct transmission mediums or carrier domains vs. Link multiplicity index. Network nodal degree = 2.6



Fig 3. Average number of distinct transmission mediums or carrier domains vs. Link multiplicity index. Network nodal degree = 3.0

If many connection requests arrive simultaneously, we can run the heuristics sequentially for each of the connection requests. If the network links have limited capacity, we may first sort the connection requests based on the length of the shortest paths between all the source-destination pairs, then apply the heuristics on the requests staring with the ones that have the longest shortest path between the source and the destination. This is because these paths are most likely to be blocked if we route them later.

To compare the performance of the heuristics, we used LEDA [25] to randomly generate networks with size ranging from 10 nodes to 40 nodes. The nodal degree ranged from 2.6 to 3.0. We also applied various values between 1 to 20 on the average number of network links that have the same transmission mediums or carrier domains, which are measured by link multiplicity index. The link multiplicity index is defined as the average number of links that have the same transmission mediums or carrier domains. If the link multiplicity index is 1, every network link has a unique transmission medium or carrier domain. The higher is the link multiplicity index, the fewer transmission mediums or carrier domains exist in the networks. To further evaluate the heuristics, we developed integer linear programming (ILP) formulation for the problem and solved them using CPLEX [26].

For every randomly generated single-path connection request, we executed the heuristics to obtain the minimum-multiplicity path. We then compare the average numbers of distinct transmission mediums or carrier domains on those paths. The lower bound is obtained from the ILP solution. To establish an upper bound, we run Dijkstra's shortest path algorithm on all the connection requests. Two sets of simulation results are shown in Figure 2 and 3. Results on other network topologies are similar.

Based on the simulation results, the paths obtained from the Single-Path Optimization Algorithm are closest to the optimal ILP solutions. This is because SPOA has a bigger pool of selection than SPRA does.

We note that, as the nodal degree increases, the number of distinct transmission mediums or carrier domains on the connection path reduces. The reason for this behavior is that an increase in nodal degree results in a wider choice of available routes for each connection request. Furthermore, an increase in nodal degree reduces the average hop distance for each connection, thereby reducing the number of distinct transmission mediums or carrier domains. We also note that, link multiplicity index has an impact on the number of transmission mediums or carrier domains on the paths as well. When the link multiplicity index is 1, every link in the network has a unique transmission medium or carrier domain; hence, the number of distinct transmission mediums or carrier domains for a given path is simply the hop count of that path. As the link multiplicity index increases, the total number of links with the same transmission medium or carrier domain increases. As a result, the number of distinct transmission mediums or carrier domains on the path decreases. The network topology and the size of the network also affect the number of distinct transmission mediums or carrier domains on the path. Larger networks with more nodes result in a higher average hop count for paths; hence, for the same nodal degree and link multiplicity index, paths in a network with more nodes have a greater number of distinct transmission mediums or carrier domains than paths in a network with fewer nodes.

4. CONCLUSION

In this paper we discussed a relatively new class of routing

problems of multimedia communication referred to as minimum-multiplicity routing problems. These problems have practical significance in applications that require finding paths satisfying various objectives such as minimizing the number of distinct transmission mediums or carrier domains. We discussed four problems and proved that these problems are NP-hard. We proposed various heuristics that execute in polynomial times and yield solutions that are very close to the optimal.

Various factors affect the number of distinct transmission mediums or carrier domains on the paths, including the nodal degree, the link multiplicity index, and the number of nodes in the network. An increase in the nodal degree helps reduce the number of distinct transmission mediums or carrier domains on the paths for the minimum-multiplicity single connection routing problem. This is due to the fact that there is a greater choice of routes for the connections. Heuristic SPOA performs the best for reducing the number of distinct transmission mediums or carrier domains on a single connection.

While the emphasis of this paper is to identify the problems and to prove their NP-hardness, the immediately future study will be developing approximation algorithms for all four problems. In addition, topologies other than single path, tree, ring and full mesh may also be investigated, together with exploration of more efficient fault-tolerance schemes.

5. **References**

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