Blocking Analysis of Multifiber Wavelength-Routed Networks

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***Abstract*— In this paper, we provide a new analytical model for evaluating the blocking performance of dynamic lightpath establishment in multifiber wavelength-routed networks. By adopting the simple link-independent model together with the wavelength correlation assumptions, we manage to achieve a good balance between analytical accuracy and computational complexity. Extensive numerical results show that the proposed model can quickly produce accurate analytical results under different traffic loads and in different networks.**

I. Introduction

Wavelength division multiplexing (WDM) has been widely deployed as a key transmission technology for providing broadband services. To fully utilize the high data rates, next-generation networks are expected to provide flexible, all-optical connections, known as the *lightpaths* [1], to the users. Lightpath-based optical networks are referred to as *wavelength-routed networks*.

In legacy backbone networks, traffic loads are generally static. Therefore the optical connections are planned offline and remain in the network semi-permanently. In next-generation optical networks, especially in the optical Internet, the data traffic loads are expected to be much more dynamic. Thus, the lightpaths need to be set up and torn down over time [2]. In such cases, multifiber wavelength-routed networks, with multiple optical fibers in each link, are becoming increasingly attractive. One reason for their attractiveness is that multifiber networks help to lower the blocking caused by the *wavelength continuity constraint*, i.e., the constraint that the same wavelength has to be used on every hop along the path when wavelength conversion is not available in the network [3]. If there are multiple fibers per link, then on each link there will be multiple channels on the same wavelength[1]. Therefore an incoming wavelength can be switched to any outgoing fiber in which the same wavelength is still available. Recent study in [4] also shows that multifiber networks with fewer wavelengths per fiber will result in lower cost compared to networks with a single fiber per link and a large number of wavelengths per fiber.

The key performance measurement in dynamic lightpath establishment is the connection blocking probability. The blocking performance of single-fiber wavelength-routed networks has been studied analytically in a number of previous work, e.g., [5]–[7]. In [5], an innovative reduced load approximation scheme with state-dependent arrival rate is developed for blocking analysis. However, this model leads to high complexity when calculating the blocking of long lightpaths. In addition, since the wavelength correlation caused by the wavelength continuity constraint is ignored, the accuracy of the model is not satisfactory in sparsely-connected networks. To overcome these problems, quite a few models that consider wavelength correlation have been developed. Among the best is the model proposed in [6], where the complexity is significantly reduced and the wavelength correlation between adjacent links is taken into consideration. In [7], the wavelength correlation model is successfully extended to investigate distributed lightpath establishment in WDM networks.

While most existing models are for single-fiber networks, blocking performance of multifiber networks is analyzed only in a few previous work [8], [9]. [9] is the latest one among these existing results that makes the link independent assumption without considering wavelength correlation. Like other models that assume noncorrelation, the model in [9] has low computational complexity, but tends to overestimate the blocking, especially in sparsely-connected networks. To the best of our knowledge, the model proposed in [8] is the only one that has considered wavelength correlation in multifiber networks. It proposes a *multifiber link-load correlation* (MLLC) model which explores the wavelength correlation by formulating the traffic loads on two adjacent links as a multidimensional Markov chain. As a result, the MLLC model achieves rather good accuracy but the complexity becomes quite high. To achieve a low computational complexity, the model in [8] adopts a non-recursive algorithm, which ignores the impact of a connection request being blocked before it could reach a certain link, while calculating the blocking probability on that link. Consequently, it tends to overestimate the traffic load on each link, and therefore overestimates the network blocking probability as well, especially under heavy traffic loads.

In this paper, we propose a new analytical model for evaluating the blocking performance of dynamic lightpath establishment in multifiber wavelength-routed networks. We adopt the link-independent model with simplified wavelength correlation assumptions. By applying a simple recursive algorithm, we

[1] In this paper, the words "channel" and "wavelength" have different meanings. Specifically, in a link with F fibers, we have F channels on each wavelength.

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manage to achieve a good balance between computational complexity and analytical accuracy. As a result, our model can efficiently produce accurate analytical results under various traffic loads in various network topologies.

The rest of this article is organized as follows. In Section II, we present the analytical model for evaluating the blocking performance of multifiber wavelength-routed networks. Numerical results and discussions are provided in Section III, and Section IV concludes this paper.

II. The Analytical Model

In this section, we propose a new analytical model for evaluating the blocking performance of multifiber wavelength-routed networks. To simplify the analysis, we make the following assumptions:

- The network has an arbitrary topology with J directional links, where each link is composed of F fibers with W wavelengths per fiber. Let $C = WF$ be the total number of channels in each link.
- There are no wavelength converters in the network.
- Connection requests between each source-destination node pair arrive as a Poisson process with an arrival rate of λ_R, where R denotes the pre-calculated *fixed* route between the source-destination nodes.
- The holding time of every connection follows the exponential distribution with an average value of $\frac{1}{\mu}$.
- A wavelength is *free* on a link if it is idle in at least one fiber along the link. Among all the wavelengths that are free along the path, one of them would be randomly selected.

A. Framework

Let X_j denote the total number of idle channels on link j and let

$$q_j(m) = \Pr[X_j = m] \qquad m = 0, 1, \cdots, C$$

be the steady state probability that there are m idle channels on link j. Following [5], [10], we assume that all the links are mutually independent. Therefore, we can define $\lambda_j(m)$ as the state dependent arrival rate of the traffic on link j and calculate all $q_j(m)$ through

$$\begin{cases} q_j(m) = q_j(0) \cdot \mu^m \cdot \prod_{k=1}^{m} \frac{(C-k+1)}{\lambda_j(k)}, & m = 1, 2, \cdots C \\ q_j(0) = \left[1 + \sum_{m=1}^{C} \mu^m \cdot \prod_{k=1}^{m} \frac{(C-k+1)}{\lambda_j(k)}\right]^{-1} \end{cases} \tag{1}$$

which is a state dependent M/M/C/C model.

We now provide a reduced-load approximation algorithm to calculate the blocking probability recursively.

Framework of the Blocking Analysis

1) Initiate $\lambda_j(m), j = 1, 2, \cdots, J$ as follows:

$$\lambda_j(m) = \begin{cases} \sum_{R:j\in R} \lambda_R, & m = 1, 2, \cdots, C \\ 0, & m = 0 \end{cases}$$

2) Calculate all $q_j(m)$ through Eq. (1).
3) Calculate the blocking probability of R as $B_R = 1 - V_R$ where V_R denotes the probability that a connection of R can be successfully set up. If for every route R, B_R has been convergent, then stop; otherwise, go to Step 4.
4) Adjust $\lambda_j(m), j = 1, 2, \cdots, J$ as follows:

$$\lambda_j(m) = \sum_{R:j\in R} \lambda_{R,j}(m) \triangleq \sum_{R:j\in R} \lambda_R \cdot V_{R|X_j=m}$$

where $\lambda_{R,j}(m)$ denotes the arrival rate of those connection requests on route R which are finally successfully accepted, given that the state of link j is m. Go to Step 2. □

It is worth noting that, although we have not been able to prove the convergency of the model, the proposed algorithm generally yields converged results in just a few iterations, as we will show later in Section III.

Hereafter we discuss how to calculate V_R. Let L_R denote the number of links on route R. To simplify the description, we denote links j and j' the j-th ($1 \leq j \leq L_R$) and the $(j-1)$-th (when $j > 1$) links on route R, respectively. We then define several parameters as follows:

- $h_R(i)$: The steady state probability that a given set of i wavelengths are free along path R;
- $g_j(i)$: The steady state probability that a given set of i wavelengths are free on link j;
- $g_{j|j'}(i)$: The conditional probability that a given set of i wavelengths are free on link j, given that this set of wavelengths are free on link j'.

Based on the principle of inclusion-exclusion, we have

$$V_R = \sum_{i=1}^{W} (-1)^{i+1} \binom{W}{i} h_R(i) \tag{2}$$

Let $N_j(i)$ denote the number of busy channels on wavelength i in link j. To keep the computation of $h_R(i)$ at a reasonable complexity, we use a wavelength correlation model similar to that in [6], [7]. Specifically, we assume that

1) All wavelengths on a certain link are identical and all channels on a given wavelength are identical. These two assumptions are reasonable since we are using the random wavelength assignment.
2) $N_j(i)$ is independent of $N_{j^*}(i)$ ($j^* \neq j, j'$) if $N_{j'}(i)$ is known.
3) $N_j(i_1)$ is independent of $N_{j'}(i_2)$ ($i_1 \neq i_2$) if $N_j(i_2)$ or $N_{j'}(i_1)$ is known.

Based on these assumptions, the $h_R(i)$ could be calculated as

$$h_R(i) = \begin{cases} g_1(i) & L_R = 1 \\ g_1(i) \cdot \prod_{j=2}^{L_R} g_{j|j'}(i) & L_R > 1. \end{cases} \tag{3}$$

In the rest parts of this section, we will elaborate on the calculations of the parameters $g_j(i)$ and $g_{j|j'}(i)$, respectively.

B. Calculation of $g_j(i)$

To calculate $g_j(i)$, we first define $g(i,m,w,F)$ as the probability that there are i free wavelengths among w ($w \le W$) wavelengths given that the total number of free channels is m and the number of fibers in the link is F. Note that this probability does not depend on any specific link, thus all $g(i,m,w,F)$'s can be pre-calculated. With pre-calculated $g(i,m,W,F)$'s, we can then derive $g_j(i)$ as

$$g_j(i) = \sum_{m=i}^{C} q_j(m) \cdot g(i,m,W,F) \qquad (4)$$

The parameter $g(i,m,w,F)$ can be calculated recursively. We first consider the boundary conditions. From the definition, we see that

$$g(i,m,w,F) = 0, \qquad \text{if } i > m \qquad (5)$$

On the other hand, if $i = 1$ and $m \ge 1$, we have a simple case where

$$g(1,m,w,F) = \begin{cases} 1 & m > (w-1)F \\ 1 - \dfrac{\binom{(w-1)F}{m}}{\binom{wF}{m}} & m \le (w-1)F \end{cases} \qquad (6)$$

Now consider the case where $1 < i \le m$. The m idle channels can be partitioned into two groups, one has k idle channels on a certain single wavelength and the other one contains $m-k$ channels on the other wavelengths. To satisfy the condition that the given set of i wavelengths are free, a valid partition must comply with the following conditions:

$$\begin{cases} 1 \le k \le F \\ i-1 \le m-k \le (w-1)F \end{cases}$$

Therefore, we can use the recursive algorithm to calculate the $g(i,m,w,F)$ as follows:

$$g(i,m,w,F) = \sum_{k=K_1}^{K_2} \frac{\binom{F}{k}\binom{(w-1)F}{m-k}}{\binom{wF}{m}} \cdot g(i-1, m-k, w-1, F) \qquad (7)$$

where

$$\begin{cases} K_1 = \max[1, m-(w-1)F] \\ K_2 = \min[F, m-(i-1)]. \end{cases}$$

C. Calculation of $g_{j|j'}(i)$

Without loss of generality, we assume that the given set of i wavelengths are indexed from 1 to i. Therefore, based on the assumptions we have made before Eq. (3), we have

$$\begin{aligned} g_{j|j'}(i) = \; & \Pr[N_j(i) < F | N_j(i-1) < F, \cdots, N_j(1) < F, N_{j'}(i) < F] \\ & \times \Pr[N_j(i-1) < F | N_j(i-2) < F, \cdots, N_j(1) < F, \\ & \qquad N_{j'}(i-1) < F] \\ & \times \cdots \\ & \times \Pr[N_j(2) < F | N_j(1) < F, N_{j'}(2) < F] \\ & \times \Pr[N_j(1) < F | N_{j'}(1) < F] \end{aligned}$$

Similar to [7], we have

$$g_{j|j'}(i) = \prod_{k=1}^{i} \left[1 + \gamma_{j'j} \times \left(\frac{1}{\eta_j(k)} - 1 \right) \right]^{-1} \qquad (8)$$

where $\eta_j(k)$ is defined as

$$\eta_j(k) = \begin{cases} g_j(1) & k = 1 \\ \dfrac{g_j(k)}{g_j(k-1)} & k > 1 \end{cases} \qquad (9)$$

and $\gamma_{j'j}$ is the wavelength correlation factor between the same wavelength on link j' and j, which is defined as

$$\gamma_{j'j} = \frac{\Pr[N_{j'}(i) < F | N_j(i) = F]}{\Pr[N_{j'}(i) < F | N_j(i) < F]} \qquad (10)$$

Note that in Eq. (10), $\gamma_{j'j}$ does not depend on i since we assume that all the wavelengths are identical. We can also observe from Eq. (8) that, if
$\gamma_{j'j} = 0$, then

$$g_{j|j'}(i) = 1,$$

which means that the same wavelength on these two links are fully correlated. On the other hand, if $\gamma_{j'j} = 1$, then

$$g_{j|j'}(i) = g_j(i)$$

which means that the same wavelength on links j' and j are uncorrelated.

To calculate the wavelength correlation factor in multifiber networks, we define the following parameters

- $y_{j'|j}(k)$: the conditional probability that there are idle channels on a given wavelength in link j', given that there are k busy channels on the same wavelength in link j;
- $z_j(k)$: the steady-state probability that there are k busy channels on a given wavelength in link j;
- $z_j(k|k < F)$: the conditional probability that there are k busy channels on a given wavelength in link j given that $k < F$;
- $u_{j'j}(k,l)$: the probability that l channels on a certain wavelength are occupied by the connections passing through both links j' and j, given that there are totally k busy channels on the same wavelength in link j.
- $\phi_{j'j}$: the conditional probability that a channel on link j is occupied by a connection passing through both links j' and j, given that the channel is busy;
- ξ_j: the probability that a given channel is busy on link j;

We can now express $\gamma_{j'j}$ as follows:

$$\gamma_{j'j} = \frac{y_{j'|j}(F)}{\sum_{k=0}^{F} y_{j'|j}(k) \cdot z_j(k|k < F)} \qquad (11)$$

In Eq. (11), $z_j(k|k < F)$ can be calculated as

$$z_j(k|k < F) = \frac{z_j(k)}{\sum_{l=0}^{F-1} z_j(l)} \qquad (12)$$

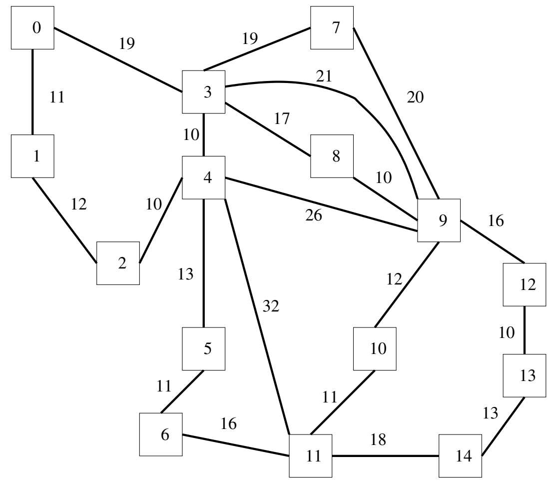

Fig. 1. A 15-node mesh network.

where

$$z_j(k) = \sum_{m=F-k}^{C-k} q_j(m) \cdot \frac{\binom{F}{F-k}\binom{C-F}{m-F+k}}{\binom{C}{m}} \quad (13)$$

To calculate the parameter $y_{j'|j}(k)$, we can first calculate $u_{j'j}(k,l)$ through

$$u_{j'j}(k,l) = \binom{k}{l} \phi_{j'j}^{l} (1-\phi_{j'j})^{k-l} \quad (14)$$

where (following [6]) $\phi_{j'j}$ can be calculated by

$$\phi_{j'j} = \frac{\sum_{R:j,j'\in R} q_j(m)\lambda_{R,j}(m)}{\sum_{R:j\in R} q_j(m)\lambda_{R,j}(m)}. \quad (15)$$

Consequently, parameter $y_{j'|j}(k)$ could be derived as follows:

$$y_{j'|j}(k) = \sum_{l=0}^{k} u_{j'j}(k,l) \times \left(1 - (\xi_{j'} \times (1-\phi_{j'j}))^{F-l}\right) \quad (16)$$

where

$$\xi_j = \sum_{m=0}^{C-1} q_j(m) \cdot \frac{C-m}{C} \quad (17)$$

D. Calculation of State Dependent Arrival Rate

To calculate the state dependent arrival rate, we apply

$$V_{R|X_j=m} = \sum_{i=1}^{\min(m,W)} (-1)^{i+1} \binom{W}{i} h_R(i|X_j=m) \quad (18)$$

where $h_R(i|X_j=m)$ can be calculated by replacing the $g_j(i)$ and $g_{j|j'}(i)$ in Eq. (3) with $g_j(i|X_j=m)$ and $g_{j|j'}(i|X_j=m)$, respectively. Note that

$$g_j(i|X_j=m) = g(i,m,W,F)$$

which has been pre-calculated. Thus we only need to calculate $g_{j|j'}(i|X_j=m)$ as follows:

$$g_{j|j'}(i|X_j=m) = \prod_{k=1}^{i} \left[1 + \gamma_{j'j} \times \left(\frac{1}{\eta_j(k|X_j=m)} - 1\right)\right]^{-1} \quad (19)$$

where

$$\eta_j(k|X_j=m) = \begin{cases} g(1,m,W,F) & k=1 \\ \dfrac{g(k,m,W,F)}{g(k-1,m,W,F)} & k>1 \end{cases} \quad (20)$$

E. Computational Complexity

We now analyze the complexity of the proposed model in each iteration.

- On an H-hop route, the most complex parameters are $(V_{R|X_j=m})$'s, which require $O(HFW^2)$ calculation.
- For the calculations of the parameters on all the links, we can observe that the complexities are upper bound by the maximum of $O(JFW^2)$, for all $g_j(i)$'s and $\eta_j(k|X_j=m)$'s, and $O(JF^2W)$, for all $z_j(k)$'s.
- To calculate all the wavelength correlation factors, we first let A denote the maximum nodal degree along the route. We can see that the most complex parameters are $g_{j|j'}(i|X_j=m)$'s, which have a complexity of $O(JAFW^2)$.

Finally, we note that the proposed model can be modified to analyze the case with different assumptions. For example, if the number of fibers per link F is not a constant in every link, only the calculations of $g_{j|j'}(i)$ needs to be changed. Due to limited space, detailed discussions on the modification method are omitted in this paper.

III. Numerical Results

In this section, we evaluate the accuracy and time efficiency of our analytical models by comparing the analysis results to the simulation results. The experiments are conducted on a 15-node mesh network (shown in Fig. 1), where the number on each link denotes the physical length in 10s of kilometers, and a 12-node bi-directional ring topology as well. We assume that there are no wavelength converters in the network. We also make the following assumptions:

- Each link has the same number of fibers and each fiber consists the same set of wavelengths.
- The traffic pattern is uniform, i.e., the arrival rate of connection requests between each source-destination node pair is identical.
- The fixed shortest-path routing is used for all the connections.

In all the numerical results, we let the traffic load, measured in Erlang, be the normalized traffic load that equals to the total load between each source-destination pair divided by the number of channels per link. All simulation results are presented with 95% confidence.

Fig. 2 shows the blocking performance of different multifiber schemes where we fix the total number of channels

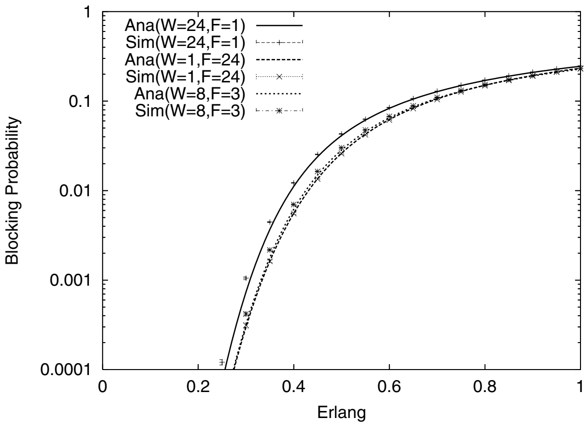

Fig. 2. Traffic load vs. network blocking in the 15-node network.

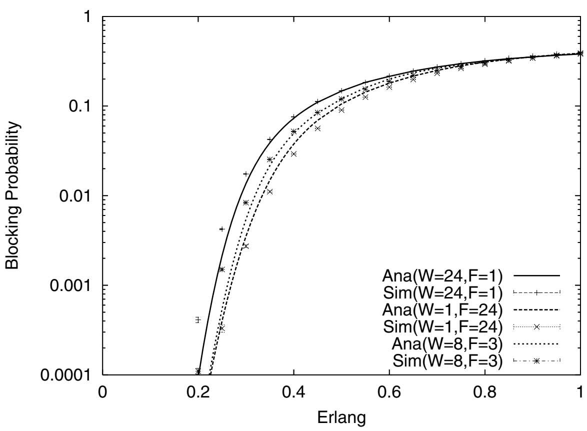

Fig. 3. Traffic load vs. network blocking in the 12-node ring.

per link as $C = 24$. Particularly, we consider three different cases: (1) $W = 24$, $F = 1$; (2) $W = 8$, $F = 3$; and (3) $W = 1$, $F = 24$. Case (1) is the typical network scenario where there is only one fiber per link. On the other hand, case (3) is equivalent to the full wavelength conversion case where an incoming connection can be switched to any outgoing channels. We observe that our analytical model is highly accurate in all the three cases under various traffic loads. In addition, in Fig. 2, it can be observed that the performance of $F = 3$ is similar to that of full conversion case.

We present the analysis and simulation results for the 12-node bi-directional ring in Fig. 3. We see that our model remains as fairly accurate. However, due to the strong wavelength correlation, the analysis results are not as accurate as those for the 15-node network, especially under light traffic loads. To further improve the analysis accuracy in such cases, more complex models would need to be developed.

Finally, Table I shows the time efficiency of the proposed analytical model. All the calculations are running on a Pentium PC with 450 MHz CPU and 128 MB memory. Calculations for the analytical model are stopped once the difference between the results of two consecutive iterations is less than 10^{-6}. Calculations for simulations are running for 10^6 connection

TABLE I

TIME EFFICIENCY OF THE PROPOSED ANALYTICAL MODEL AND THE EFFECTS OF F, IN THE 15-NODE NETWORK ($\rho = 0.4$ ERLANG).

		Analysis			Simulation	
F	W	Iteration	Time (s)	Blocking	Time (s)	Blocking
1	32	6	0.437	5.61E-3	53.8	6.51±0.15E-3
2	16	5	0.267	3.37E-3	48.8	4.02±0.10E-3
4	8	4	0.142	2.76E-3	46.3	3.07±0.06E-3
8	4	4	0.095	2.64E-3	45.0	2.83±0.07E-3
16	2	3	0.066	2.62E-3	44.2	2.69±0.08E-3
32	1	3	0.062	2.62E-3	43.9	2.69±0.08E-3

requests. We see that the analytical model always converges in less than one second, after only several iterations. The time consumption is much shorter than that of the numerical simulation. In addition, we see that having four fibers per link can generally achieve nearly the same performance as that of the full wavelength conversion case.

IV. CONCLUSIONS

In this paper, we proposed a new theoretical model for analyzing blocking performance in multifiber wavelength-routed networks. By using the simple link-independent model with proper wavelength correlation assumptions, a good balance between complexity and accuracy is achieved. The good matching with the extensive numerical results demonstrates the accuracy and the efficiency of the proposed model under different traffic loads in different networks.

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