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Dynamic lightpath protection in WDM mesh networks under wavelength-continuity and risk-disjoint constraints ☆

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Abstract

Path protection requires finding a working path and a protection path that are link disjoint. In this paper, we consider two fundamental problems on dynamic lightpath protection in WDM mesh networks. In the first problem, we consider a network without wavelength converters; thus both the working lightpath and protection lightpath are subject to the wavelength continuity constraint. Existing polynomial time algorithms can be applied to find a pair of link-disjoint lightpaths on a single wavelength; however, such algorithms fail if the working and protection lightpaths are on two different wavelengths. In the second problem, we consider a network with full wavelength conversion; thus the wavelength continuity constraint does not apply. Yet a single factor can cause multiple fiber links to fail simultaneously. The problem becomes finding link-disjoint lightpaths that are also risk disjoint. We prove that both of the two problems are NP-complete. We develop ILP formulations and heuristic algorithms for the two NP-complete problems. Practical constraints such as service level agreement (SLA) and priority are also considered. Computer simulations are conducted to evaluate the performance of the heuristic algorithms.

Keywords: Path protection; Wavelength continuity constraint; Shared risk link group; Risk disjoint; Wavelength-division multiplexing (WDM) networks

1. Introduction

The rapid growth of the Internet and the WWW are demanding more bandwidth from network carriers, pushing them to deploy high speed networks that use wavelength division multiplexing (WDM)

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technologies. A wavelength routed WDM network enables end users to communicate with one another via all-optical WDM channels that may span multiple nodes and fiber links. Such all-optical channels are referred to as lightpaths [1,2].

Once an end-to-end lightpath is established, a node or a fiber link failure may lead to the failure of all lightpaths that traverse that failed node or fiber link. Since the aggregate data rate on a single WDM fiber link can be as high as 50 Tbps, a failure may potentially lead to the loss of a large amount of data. To minimize the data loss, we need to develop appropriate protection and restoration schemes [3,4].

In protection schemes, backup resources are pre-computed and reserved for each connection before a failure occurs [5,6]. In restoration schemes, a route and free wavelength are discovered dynamically for each interrupted connection after a failure occurs [7,8]. A restoration scheme is usually more resource efficient, while a protection scheme has a faster recovery time and provides guaranteed recovery ability. Therefore protection schemes are more suitable for mission-critical applications. We consider protection schemes in this paper.

Protection schemes can be divided into two categories based on the level of network resource involved in the protection. These two categories are path protection and link protection. In path protection, two disjoint lightpaths are established for each connection: a working lightpath and a protection lightpath. Under normal operation, user traffic is carried on the working lightpath, but in the case of a failure on the working lightpath, the traffic is switched to the pre-reserved protection lightpath. In link protection, separate backup resources are reserved for each individual fiber link along the working lightpath. When a fiber link fails, the traffic is rerouted only around the failed link. Path protection usually has lower resource requirements and lower end-to-end propagation delay for the recovered route, while link protection can provide faster recovery since no end-to-end signaling is required [5,7].

Protection schemes can be further divided into two categories based on whether backup resources are shared by more than one connection. The first category is dedicated protection. In this type of protection, no sharing of backup resources is allowed; thus each fiber link or node can be reserved as a backup resource for at most one connection. Examples of dedicated protection are 1 + 1 protection and 1:1 protection as in SONET [9]. In 1 + 1 protection, traffic is carried simultaneously on both the working path and the protection path. In 1:1 protection, traffic is carried only on the working path until a failure occurs to the working path and traffic is switched to the protection path. The second category is shared protection. In this type of protection, a fiber link or node can be reserved as a backup resource for multiple connections, as long as those connections do not fail simultaneously. One examples of shared protection is 1:N protection in SONET, where one protection path is shared by N working paths. Dedicated protection requires more network resource but less signaling and management, while shared protection is more resource efficient but requires complex signaling and management [2].

The lightpath protection problem can be considered under either static or dynamic traffic. Under static traffic, the set of all connection requests is known in advance. The working and protection lightpaths for each connection request must be routed, and a wavelength must be assigned for each lightpath. It has been shown in [10] that the related problem of finding disjoint lightpaths for a collection of k source-destination pairs is NP-complete. The problem can be formulated using Integer Linear Programming (ILP) and can be solved through heuristic algorithms [5,11–13]. Under dynamic traffic, connection requests arrive one at a time and each connection exists for only a finite duration, referred to as the connection holding time. Subsequently, routing the working lightpath and protection lightpath is done individually for each connection request. Once a connection departs, all the network resources used by the working lightpath and the protection lightpath are released and become available to new connections.

Connection requests handled by transport network carriers have traditionally been static. A customer normally requests one or more end-to-end virtual circuits and leases them for months or years. During the lease period, even when the customer does not have traffic to send over the network, the resources reserved for that customer may not be shared by other customers. On the other hand, if a network supports dynamic traffic, a customer can reserve network resources for only the duration that it is actually needed. Thus the network utilization is improved. Emerging applications such as grid computing [14], video conferencing, and video-on-demand suite well with this dynamic traffic model. Major efforts are now under way in the IETF CCAMP working group and the ITU-T to standardize GMPLS [15], which makes it feasible to dynamically establish and release a lightpath. Therefore, we consider the lightpath protection problems under dynamic traffic in this paper.

For WDM networks, we use "link" and "fiber link" interchangeably in this paper. A lightpath traverses one wavelength on each intermediate fiber link. A link failure such as a fiber cut causes all the lightpaths on that link to fail. Compared to link failures, node failures are often much less of a concern because modern network node devices usually have built-in redundancies which greatly reduce node failure. In addition, node failure problems can often be transformed into multi-link failure problems. Therefore, we consider lightpath protection against link failure in this paper.

For lightpath protection under dynamic traffic, the fundamental problem is to find a pair of link-disjoint paths from a source node s to a destination node d. In a WDM mesh network without wavelength conversion capability [16,17], the establishment of a working lightpath and its protection lightpath is subject to the wavelength continuity constraint, i.e., a lightpath is required to be on the same wavelength channel throughout its entire path in the network. The working lightpath and its protection lightpath may both be on the same wavelength, or each may be on a different wavelength. We may intuitively attempt to find the two lightpaths using a simple two-step solution. In this approach, the first step is to find the minimum cost path from the source to the destination, and to let this path be the working lightpath. The second step is to remove all links on the working lightpath and to find another minimum cost path from the source to the destination. If the second path is found, it is guaranteed to be link disjoint from the working lightpath, and it is designated as the protection lightpath. However, this two-step solution may not yield valid disjoint lightpaths for some network topologies, even though the lightpaths do exist. An example of such a network is depicted in Fig. 1. This network has only one wavelength. The simple two-step solution finds the first minimum cost path from source node s to destination node d along lightpath s–a–b–d but fails to find a second link-disjoint lightpath, even though two link-disjoint lightpaths exist (s–e–b–d and s–a–f–d).

For single-wavelength networks, a feasible solution can be found using Suurballe's algorithm and its variations [18,19], Appendix A. The total cost of the resulting two link-disjoint lightpaths is minimal among all such path pairs. The algorithm runs in $O(n^2 \log n)$ time, where n is the number of nodes. For networks with multiple wavelengths, the network is equivalent to a stack of parallel networks, each on a different wavelength. We can apply Suurballe's algorithm on each of the parallel networks in order to find a pair of link-disjoint lightpaths from the source to the destination on the same wavelength. However, if such lightpaths do not exist on the same wavelength, Suurballe's algorithm may fail. An example is given in Fig. 2.

In this example, a WDM network of two wavelengths without wavelength converter is visualized as two parallel networks of wavelength λ_1 and λ_2 stacked together. Wavelength λ_1 on fiber links sa and fd is unavailable and wavelength λ_2 on fiber link eb is unavailable. Although there exists one pair of link-disjoint paths (i.e., the thick solid lines)

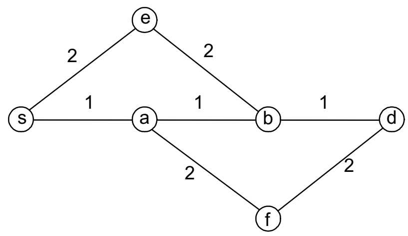

Fig. 1. A single-wavelength WDM network. The numbers indicate link costs.

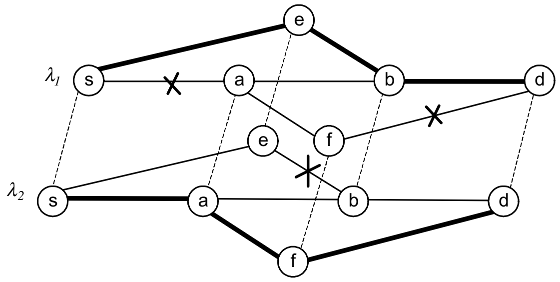

Fig. 2. A two-wavelength WDM. There exists only one pair of link-disjoint paths (i.e., the thick solid lines) from s to d, one on λ_1 and the other on λ_2.

from s to d, one on λ_1 and the other on λ_2, Suurballe's algorithm may not to find them. Therefore, for WDM network without wavelength converters, lightpath protection often requires finding two link-disjoint lightpaths on two different wavelengths. It is known that, for the special case in which the total cost of the two lightpaths is to be minimal, the problem is NP-complete [20]. However, a more fundamental question is whether the dynamic lightpath protection problem is solvable without the total cost constraint. We will prove that the problem is still NP-complete for both dedicated protection and shared protection, regardless of the total path costs.

Several heuristic techniques have been proposed to solve the Routing and Wavelength Assignment (RWA) problem of lightpath protection, which can be applied here. In [21], K alternate link-disjoint routes are predefined for each s–d pair. When a connection request arrives, all wavelengths on the K routes are examined until a pair of link-disjoint lightpaths with the minimal total cost is found. An improved heuristic is introduced in [22]. This heuristic predefines two groups of alternate shortest routes for each s–d pair: M routes for the working path and B routes for the protection path. Routes within the same group are not necessarily link-disjoint from each other, but a route in one group is link-disjoint from all routes in the other group. When a connection request arrives, all routes in the working path group are searched until a route is found with a free wavelength on the entire route. Similarly, all routes in the protection path group are searched until an available route is found. Since the set of working and protection routes is predefined, both heuristics are not adaptive to the dynamic status of the network, which leads to high blocking probability for incoming connection requests as traffic load increases. Another issue with the two heuristics is that the average number of the fixed alternate paths is limited by the nodal degree of the network, which further reduces their adaptability. In this paper, we develop heuristics that are fully adaptive to real time network status.

Now consider the lightpath protection problem in a WDM network with full wavelength conversion capability at every optical switch. The wavelength conversion capability eliminates the wavelength continuity constraint on lightpaths, and we can now apply Suurballe's algorithm to find a working lightpath and its link-disjoint protection lightpath. However, these two lightpaths may still fail simultaneously if a single factor can cause more than one link failures, and the failed fiber links happen to be on both the working and protection lightpaths.

In a special case of this type of network configuration, multiple fiber links are bundled into the same underground conduit, or span. Even though these fiber links are disjoint in the network layer, a cut to the underground conduit can cause all the fiber links to fail. To describe this type of network configuration, transport network carriers use the notation of Shared Risk Link Group (SRLG) [23,24]. Those fiber links in the same conduit belong to the same SRLG because they all share the same risk factor of a conduit cut. In order for path protection to work, all the fiber links on a protection path must be in different SRLGs from those links on the working path. Therefore, in addition to being link disjoint, the protection routing problem in this type of network has the extra constraint of being SRLG disjoint or risk disjoint.

It has been reported that a large network may contain several hundred SRLGs. Finding link-disjoint paths with the additional SRLG-disjoint constraint is therefore more complex than finding merely link-disjoint paths. It has been shown that for some special SRLG configurations, such as forks and express links, there still exist algorithms with polynomial time complexity [19,25]. These algorithms use graph transformations techniques such that the special SRLG configurations can be treated as regular links and nodes. Yet when the configurations are arbitrary, algorithms with polynomial time complexity have not been found. For instance, in a so-called "bridge" configuration as shown in Fig. 3, $m(m > 1)$ fiber links share the same span; thus, the fibers are in the same SRLG. Trying to find two SRLG- disjoint paths in a network containing such a configuration is likely to have $O(2^m)$ complexity [19].

The concept of SRLG also applies to WDM wavelength-routed networks where the same risk factor may take down multiple lightpaths. Using 3-SAT reduction, [26] first proved that the problem of finding two risk-disjoint lightpaths is NP-complete if the risks are arbitrarily distributed. It is possible to use the Set-Splitting reduction for the proof, too. In this paper, we give a much simpler proof based on the NP-completeness of the previous problem under the wavelength continuity constraint. To extend the result and the concept of SRLG beyond the scope of fiber span failure, we introduce the concepts of *Risk ID* and *Risk Set* for describing risk distributions in general connection-oriented networks.

Heuristics in [27] and [28] use a simple two-step approach to find risk-disjoint paths. These heuristics assign a higher cost to fiber links with Risk IDs that occur more frequently in the network, so that such links are less likely to be selected by a working path. As a result, it is more likely that a risk-disjoint protection path can be found. In [13], a heuristic is proposed for the special case of path protection under the duct-layer constraint. The heuristic first applies Suurballe's algorithm in the duct layer to find two duct-disjoint paths, then assigns free fiber links to the paths. In this work, we develop heuristic algorithms for the general case of the problem in which risks are arbitrarily distributed in the network.

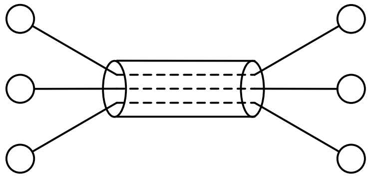

Fig. 3. Illustration of a "bridge" configuration. The three parallel fiber links are in the same SRLG.

In addition to the wavelength continuity constraint and the risk-disjoint constraint, in practice, a connection request may have additional constraints such as service level agreement (SLA) and priority [29]. One important SLA parameter is the maximum end-to-end delay. It applies to both the working lightpath and the protection lightpath. For lightpaths in a WDM network, end-to-end delays come primarily from the propagation delays on fiber links, therefore the maximum delay can be translated into the maximum length of the lightpaths. As another practical constraint, priority allows a network to give precedence to a connection request of high priority over a connection request of low priority if both requests arrive simultaneously at the network. On the other hand, a connection of high priority often requires a working lightpath and its protection lightpath to be completely link disjoint and risk disjoint, while a connection of lower priority may allow the working lightpath and the protection lightpath to share common links or risks. In this work, we consider the dynamic lightpath protection problems under these additional practical constraints as well.

To the best of our knowledge, our work is the first to prove the NP-completeness of the general case of the dynamic lightpath protection problem under the wavelength continuity constraint. The heuristics we develop for this problem are the first to be fully adaptive to the real-time network status. While an NP-complete proof of the dynamic lightpath protection problem under the risk-disjoint constraint already exists, our proof is much simpler. We are also the first to propose the concept of *Risk ID* and *Risk Set* so that we can expand some of our results on optical networks to general connection-oriented networks. The heuristic solution we propose for this problem is among the first to address network topologies with arbitrary risk configuration. The solution is much superior to existing approaches.

The rest of the paper is organized as follows. Section 2 investigates the problem of finding a working lightpath and its protection lightpath under the wavelength continuity constraint. We prove that the problem is NP-complete if the two lightpaths are on different wavelengths, regardless of the total lightpath costs. We then develop ILP formulations and heuristic solutions for the problem. Section 3 addresses the problem of finding a working lightpath and its protection lightpath under the risk-disjoint constraint. We give a simple proof of the NP-completeness of the problem. We then extend the result to general connection-oriented networks. We develop ILP formulations and heuristic solutions for this problem. In Section 4, we conduct computer simulations on the heuristic solutions and compare their performance. Section 5 concludes the paper.

2. Dynamic lightpath protection under the wavelength continuity constraint

In WDM networks, if the optical switches do not have wavelength conversion capability, lightpaths are subject to the wavelength continuity constraint, i.e., a lightpath is required to be on the same wavelength channel throughout its entire path in the network. This constraint applies to the establishments of both the working lightpath and the protection lightpath, even though the working lightpath and the protection lightpath may each be on a different wavelength. Suurballe's algorithm can be used to determine whether a working lightpath and its protection lightpath exist on a single wavelength. If the algorithm fails, then we need to find two link-disjoint lightpaths, each on a different wavelength. We now prove that this problem is NP-complete.

2.1. Proof of NP-completeness for dedicated protection

The problem is formally defined as follows. Given optical network $G = (N, L)$, where N is the set of optical switching nodes and L is the set of fiber links, and given the number of wavelengths on each link, find two link-disjoint lightpaths from source node s to destination node d such that each lightpath is on a different wavelength, and neither lightpath shares common wavelength with existing lightpaths on the same links (i.e., dedicated protection).

We reduce the 3SAT problem, which is known to be NP-complete [30,31], Appendix B, to the target problem. The 3SAT problem is stated as follows. Given a collection $C = \{C_1, C_2, \ldots, C_M\}$ of clauses on a finite set $V = \{v_1, v_2, \ldots, v_N\}$ of variables such that $|C_j| = 3$ for $1 \leqslant j \leqslant M$, where clause C_j is the boolean "or" of three literals (a literal is either a variable or the boolean "not" of a variable) and is satisfied by a truth assignment if and only if at least one of the three literals is true, is there a truth assignment for V that satisfies all the clauses in C?

We construct a graph G for an arbitrary instance of 3SAT C, such that the graph contains two link-disjoint lightpaths, P_1 on wavelength λ_1 and P_2 on wavelength λ_2, from node s to node d if and only if there is a truth assignment satisfying all clauses. In this proof, the graph contains only two wavelengths, λ_1 and λ_2, but it can be easily expanded to the case of more wavelengths. Following are the steps for the graph construction:

1. Create source node s and destination node d.
2. Corresponding to the N variables in V, create $n+1$ nodes z_i, $0 \leqslant i \leqslant N$. There is a link from s to z_0 and from z_N to d. Between z_{i-1} and z_i, there are nodes $x_i^1, y_i^1, x_i^2, y_i^2, \ldots, x_i^M, y_i^M$, and $\bar{x}_i^1$, $\bar{y}_i^1$, $\bar{x}_i^2$, $\bar{y}_i^2, \ldots, \bar{x}_i^M, \bar{y}_i^M$, which correspond to the M clauses in C. There are links $z_{i-1}x_i^1, x_i^1y_i^1$, $y_i^1x_i^2$, $x_i^2y_i^2, \ldots, x_i^My_i^M$, $y_i^Mz_i$ and links $z_{i-1}\bar{x}_i^1$, $\bar{x}_i^1\bar{y}_i^1$, $\bar{y}_i^1\bar{x}_i^2$, $\bar{x}_i^2\bar{y}_i^2, \ldots, \bar{x}_i^M\bar{y}_i^M$, $\bar{y}_i^Mz_i$. Links $x_i^jy_i^j$ and $\bar{x}_i^j\bar{y}_i^j$ each contain two wavelengths, λ_1 and λ_2. All other links created in this step contain only wavelength λ_1.
3. Corresponding to each clause C_j, create nodes u_j and w_j, $1 \leqslant j \leqslant M$. There is a link from s to u_1 and from w_M to d. There is also a link from w_j to u_{j+1}. Other links are formed according to the following rules:
   a. A link from u_j to x_i^j exists, and a link from y_i^j to w_j exists, if and only if variable v_i is in clause C_j.

b. A link from u_j to $\bar{x}_i^j$ exists, and a link from $\bar{y}_i^j$ to w_j exists, if and only if variable $\bar{v}_i$ is in clause C_j.
   All links constructed in this step only contain wavelength λ_2.

An example is given in Fig. 4. In this example, we construct graph G for a 3SAT instance $C = \{C_1, C_2\}$, $V = \{v_1, v_2, v_3\}$, $C_1 = v_1 \vee \bar{v}_2 \vee v_3$, $C_2 = \bar{v}_1 \vee v_2 \vee v_3$. The dotted links contain wavelength λ_1 and the dashed links contain wavelength λ_2. The solid links contain both wavelengths λ_1 and λ_2. For a truth assignment $v_1 = 1$, $v_2 = 1$, $v_3 = 1$, the corresponding disjoint paths are: $p_1(s - z_0 - \bar{x}_1^1 - \bar{y}_1^1 - \bar{x}_1^2 - \bar{y}_1^2 - z_1 - \bar{x}_2^1 - \bar{y}_2^1 - \bar{x}_2^2 - \bar{y}_2^2 - z_2 - \bar{x}_3^1 - \bar{y}_3^1 - \bar{x}_3^2 - \bar{y}_3^2 - z_3 - d)$ on wavelength λ_1 and $p_2(s - u_1 - x_1^1 - y_1^1 - w_1 - u_2 - x_2^2 - y_2^2 - w_2 - d)$ on wavelength λ_2.

Lemma 1. *If C is satisfiable, then there exist two link-disjoint lightpaths of different wavelengths from node s to node d in graph G.*

Proof. Let boolean values be assigned to $v_1, v_2, \ldots, v_N$ that satisfy C. The two lightpaths should be routed as follows:

- P_1 is on wavelength λ_1. It traverses all z_i nodes for $0 \leqslant i \leqslant N$. Between node z_{i-1} and z_i, the lightpath is routed via nodes x_i^j and y_i^j $(1 \leqslant j \leqslant M)$ if and only if $v_i = 0$. Otherwise it is routed via nodes $\bar{x}_i^j$ and $\bar{y}_i^j$.
- P_2 is on wavelength λ_2. It traverses all u_j, w_j nodes for $1 \leqslant j \leqslant M$. Between node u_j and w_j, the lightpath is routed as follows. By construction, link u_jw_j corresponds to clause C_j which has three literals. Each of the literals corresponds to a path from u_j to w_j that goes either through nodes x_i^j and y_i^j if the literal is in the form of v_j, or through nodes $\bar{x}_i^j$ and $\bar{y}_i^j$ if the literal is in the negation form, $\bar{v}_j$.

Because C is satisfied, at least one of the three literals in C_j must be 1. Let the variable in that true literal be v_j. Then

- if the literal is in the form of v_j, then $v_j = 1$, and let P_2 passes through nodes x_i^j, y_i^j;
- if the literal is in the form of $\bar{v}_j$, then $v_j = 0$, and let P_2 passes through nodes $\bar{x}_i^j$, $\bar{y}_i^j$.

If more than one literal is true, then randomly pick one of the true literals and route P_2 accordingly.

Thus, P_1 does not traverse any of the nodes u_j, w_j for $1 \leqslant j \leqslant M$, and P_2 does not traverse any of the nodes z_i for $0 \leqslant i \leqslant N$. Furthermore, if P_2 traverses node x_i^j, y_i^j, then P_1 traverses $\bar{x}_i^j$, $\bar{y}_i^j$, and vice versa. Therefore P_1 and P_2 are link disjoint, and each is on a different wavelength. □

Lemma 2. *If there exist two link-disjoint lightpaths of different wavelengths from s to d in the constructed graph G, then C can be satisfied.*

Proof

1. Since there are only two links originating from the source node s, the two links must each belong to a separate lightpath. Let sz_0 be part of P_1 and su_1 be part of P_2.

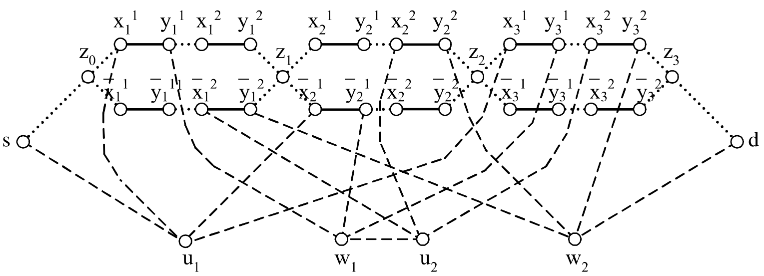

Fig. 4. A graph constructed from a 3SAT instance.

2. Since P_2 is already on wavelength λ_2, P_1 must not traverse any of the nodes u_j, w_j for $1 \leqslant j \leqslant M$, otherwise it would also be on wavelength λ_2 and violate the wavelength continuity constraint. Therefore, if P_1 traverses x_i^1 for $1 \leqslant i \leqslant N$, then it must also traverse y_i^1, $x_i^2, y_i^2, \ldots, x_i^M, y_i^M, z_i$. Similarly if P_1 traverses $\bar{x}_i^1$ for $1 \leqslant i \leqslant N$, then it must also traverse, $\bar{y}_i^1, \bar{x}_i^2, \bar{y}_i^2, \ldots, \bar{x}_i^M, \bar{y}_i^M, z_i$.
3. Since P_1 is already on wavelength λ_1, P_2 must not traverse any of the nodes z_i for $0 \leqslant i \leqslant N$, otherwise it would also be on wavelength λ_1 and violate the wavelength continuity constraint. Furthermore, if P_2 traverses node u_j $(1 \leqslant j \leqslant M)$ and x_i^j $(1 \leqslant i \leqslant N)$, it must also traverse y_i^j and then back to w_j. Similarly, if P_2 traverses node u_j and $\bar{x}_i^j$, it must also traverse $\bar{y}_i^j$ and then back to w_j.
4. Loops are not allowed. Therefore once P_2 reaches w_j $(1 \leqslant j \leqslant M)$, it must go to u_{i+1} if $j < M$, or to d if $j = M$.
5. If P_2 traverses nodes x_i^j, y_i^j, $1 \leqslant j \leqslant M$, $1 \leqslant i \leqslant N$, it must not also traverse nodes $\bar{x}_i^k$ and $\bar{y}_i^k$, $k \neq j$, and vice versa; otherwise P_1 is "blocked" and cannot reach the destination node d without violating the link disjoint constraint.
6. If P_2 traverses nodes x_i^j, y_i^j, $1 \leqslant j \leqslant M$, $1 \leqslant i \leqslant N$, then P_1 must traverses nodes traverses $\bar{x}_i^1$ for $1 \leqslant i \leqslant N$, then it must also traverse $\bar{x}_i^1$, $\bar{y}_i^1$, $\bar{x}_i^2$, $\bar{y}_i^2, \ldots, \bar{x}_i^j$, $\bar{y}_i^j, \ldots, \bar{x}_i^M$, $\bar{y}_i^M$. Similarly if P_2 traverses nodes $\bar{x}_i^j \bar{y}_i^j$, then P_1 must traverses nodes $x_i^1, y_i^1, x_i^2, y_i^2, \ldots, x_i^j, y_i^j, \ldots, x_i^M, y_i^M$.
7. Assign values to $v_1, v_2, \ldots, v_N$ as follows:
   - If P_2 traverses nodes x_i^j, y_i^j, $1 \leqslant j \leqslant M$, $1 \leqslant i \leqslant N$, then assign $v_i = 1$, making clause C_j to be true.
   - If P_2 traverses nodes $\bar{x}_i^j$, $\bar{y}_i^j$, $1 \leqslant j \leqslant M$, $1 \leqslant i \leqslant N$, then assign $v_i = 0$, making clause C_j to be true.
   - Variables that are not assigned a value in the first two steps are randomly assigned either 1 or 0.
   - This assignment satisfies C. □

Combining Lemmas 1 and 2, we see that the 3SAT problem is reducible to the problem of finding disjoint lightpaths on different wavelengths with dedicated protection. Therefore this problem is NP-complete, regardless of the paths costs.

2.2. Proof of NP-completeness for shared protection

When shared protection is allowed, the protection lightpath may share common wavelength with at most T $(T \geqslant 0)$ existing protection lightpaths on the same links. We prove the problem to be NP-complete by contradiction. If the problem with shared protection is solvable, then the problem with dedicated protection can also be solved since it is a special case of that with shared protection.

2.3. ILP formulations

In this section, we develop an ILP formulation for the dynamic lightpath protection problem under the wavelength continuity constraint. We also develop two ILP formulations for the problem with additional SLA and priority constraints. Since the traffic is dynamic, an ILP formulation should be solved for each incoming connection request.

2.3.1. ILP formulation for the problem under only the wavelength continuity constraint

The first ILP formulation is for the connection requests without additional SLA or priority constraints. The following are given as inputs to the problem:

- N: number of nodes in the network,
- L: collection of all fiber links in the network,
- Λ_{ij}: collection of all free wavelengths on fiber link $ij \in L$. Λ_{ij} is empty if all wavelengths on link ij are already taken by previously established lightpaths,
- W: the maximum number of wavelengths on any fiber link,
- s: source node,
- d: destination node.

The ILP solves for the following variables:

- α_{ij}^{sdw}: 1 if wavelength w on link ij is taken by the working lightpath from source s to destination d; 0 otherwise.

- β_{ij}^{sdw}: 1 if wavelength w on link ij is taken by the protection lightpath from source s to destination d; 0 otherwise.

Objective: Find a working lightpath and a protection lightpath that satisfy the wavelength continuity constraint.

$$\sum_{\forall ij \in L} \sum_{\forall w \in \Lambda_{ij}} \alpha_{ij}^{sdw} + \sum_{\forall ij \in L} \sum_{\forall w \in \Lambda_{ij}} \beta_{ij}^{sdw} > 0 \tag{2.1}$$

Constraints:
Flow-conservation under the wavelength continuity constraint:

$$\sum_{i=1}^{N} \alpha_{il}^{sdw} - \sum_{j=1}^{N} \alpha_{lj}^{sdw} = \begin{cases} 1, & \text{if } l = d \\ -1, & \text{if } l = s \\ 0, & \text{otherwise} \end{cases} \quad 1 \leqslant l \leqslant N,\ 1 \leqslant w \leqslant W \tag{2.2}$$

$$\sum_{i=1}^{N} \beta_{il}^{sdw} - \sum_{j=1}^{N} \beta_{lj}^{sdw} = \begin{cases} 1, & \text{if } l = d \\ -1, & \text{if } l = s \\ 0, & \text{otherwise} \end{cases} \quad 1 \leqslant l \leqslant N, \quad 1 \leqslant w \leqslant W \tag{2.3}$$

Link disjoint constraint by the problem definition:

$$\sum_{\forall w \in \Lambda_{ij}} \alpha_{ij}^{sdw} + \sum_{\forall w \in \Lambda_{ij}} \beta_{ij}^{sdw} \leqslant 1,\ \forall ij \in L \tag{2.4}$$

2.3.2. ILP formulation for the problem under additional SLA and high priority constraints

The second ILP formulation is for the connection requests with a high priority that requires strict link disjointness of the working lightpath and the protection lightpath, and an SLA constraint on the maximum delay on both lightpaths. The following are given as inputs to the problem:

- All the inputs in the first ILP formulation,
- $\langle d_{ij} \rangle$: Propagation delay on fiber link ij, $\forall ij \in L$,
- D: The maximum delay for both lightpaths. The ILP solves for the following variables:
- Same as those in the first ILP formulation.

Objective: Same as (2.1)
Constraints:
Constraints (2.2), (2.3), (2.4)

$$\sum_{\forall ij \in L} \sum_{\forall w \in \Lambda_{ij}} d_{ij} \alpha_{ij}^{sdw} \leqslant D \tag{2.5}$$

$$\sum_{\forall ij \in L} \sum_{\forall w \in \Lambda_{ij}} d_{ij} \beta_{ij}^{sdw} \leqslant D \tag{2.6}$$

2.3.3. ILP formulation for the problem under additional SLA and low priority constraints

The third ILP formulation is for the connection requests with the additional SLA constraint on the maximum delay, and a low priority that allows link sharing by the working lightpath and the protection lightpath. The following are given as inputs to the problem:

- All the inputs in the second ILP formulation. The ILP solves for the following variables:
- Same as those in the first ILP formulation.
- γ_{ij}^{sd}: 1 both the working lightpath and the protection lightpath go through link ij (on different wavelengths); 0 otherwise.

Objective: Find a working lightpath and a protection lightpath that share the minimum number of fiber links.

$$\text{Minimize} \quad \sum_{\forall ij \in L} \gamma_{ij}^{sd} \tag{2.7}$$

Constraints:
Constraints (2.2), (2.3), (2.5), (2.6)
Link sharing but not wavelength sharing:

$$\sum_{\forall w \in \Lambda_{ij}} (\alpha_{ij}^{sdw} + \beta_{ij}^{sdw}) \leqslant 1, \quad \forall ij \in L \tag{2.8}$$

$$0 \leqslant \sum_{\forall w \in \Lambda_{ij}} \alpha_{ij}^{sdw} + \sum_{\forall w \in \Lambda_{ij}} \beta_{ij}^{sdw} - 2 \cdot \gamma_{ij}^{sd} \leqslant 1, \quad \forall ij \in L \tag{2.9}$$

2.4. Heuristic algorithms

In this section we introduce heuristic algorithms for finding link-disjoint lightpaths in

WDM networks under the wavelength continuity constraint. The first algorithm is named the Route-First Algorithm. In this algorithm, we first use Suurballe's algorithm to find two link-disjoint routes with the minimum total cost, and then assign free wavelengths to them. The second algorithm is named the Wavelength-Scan Algorithm. In this algorithm, we first scan through each wavelength for a pair of link-disjoint lightpaths on the same wavelength using Suurballe's algorithm. If this step fails, we use a two-step approach and search through each pair of different wavelengths for two links-disjoint lightpaths on different wavelengths. To increase resource utilization and thus reduce blocking probability, we later modify these two algorithms to support shared protection.

The details of the algorithms are as follows. In the Route-First Algorithm, we follow three steps:

1. Scan all fiber links and increase the cost of a link linearly to the number of wavelengths already in use on the link. This step enables the Suurballe's algorithms to avoid links with fewer free wavelengths and thus increases the success possibility in Step 3. It also balances traffic load among the network links, thus improves the blocking probability.
2. Run Suurballe's algorithm and obtain two link-disjoint routes from the source to the destination with the minimum total cost.
3. Assign a free wavelength to each of the two routes if a free wavelength is available on the entire route. The wavelength assignment is done using either the First-Fit scheme, i.e., we assign the first free wavelength we can find to a lightpath, or other schemes such as Best-Fit or Minimum-Load [32].

A connection request is blocked if Suurballe's algorithm fails to find two link-disjoint routes, or if there are no free wavelengths on either route. The running time is $O(n^2 \log n + Wn)$, where n is the number of nodes and W is the number of wavelengths in the network. The pseudo code is given in Fig. 5. Note that the pseudo code only serves to illustrate the basic ideas. All modifications made to the network topology are temporary and apply only within the scope of the local procedures.

```
initialize c_l to the cost of link l  for all fiber links;
for ( all network links)
  increase c_l on link l linearly to the number of wavelengths in
  use on l;

//Searching for two link disjoint routes from s to d
if ( Suurballe's algorithm(s, d) finds two routes r_1 and r_2)
{
  //We use First-Fit scheme for wavelength assignment.
  //An alternative is Best-Fit or Minimum-Load scheme.
  for ( all wavelengths λ_i and  λ_j  in the network and λ_i ≠ λ_j )
  {
    if (λ_i is available on route r_1 AND λ_j is available on route r_2 )
    {
       assign λ_i  to  r_1 and make it the working lightpath p_1;
       assign λ_j to r_2 and make it the protection lightpath p_2;
       return( p_1 and p_2);
    }
  }
}
return(FAILURE);
```

Fig. 5. Route-First Algorithm.

In the Wavelength-Scan Algorithm, we first run Suurballe's algorithm on every wavelength to find the working lightpath and the protection lightpath on a single wavelength. We choose the wavelength for which the pair of lightpaths has the minimal total cost. If this step fails, we invoke the simple two-step algorithm as described in Section 1 on all wavelengths, i.e., run Dijkstra's algorithm to get a minimum cost path, then remove all the links on that path and run Dijkstra's algorithm again to get a link-disjoint path. The pseudo code is given in Fig. 6.

This algorithm has a running time of $O(W \cdot n^2 \log n)$, where n is the number of nodes and W is the number of wavelengths in the network.

Comparing the two algorithms, the Route-First Algorithm obtains two link-disjoint routes first before it assigns free wavelengths to them. If the algorithm returns successfully, the total cost of the two lightpaths is minimal among all link-disjoint paths from s to d. On the other hand, the Wavelength-Scan Algorithm scans through all available wavelengths, searching for the two link-disjoint paths, first on a single wavelength then on different wavelengths. Thus the running time of this algorithm is higher. When the traffic load

```
initialize c_l to the cost of link l for all fiber links;
for ( all network links)
    increase c_l on link l linearly to the number of wavelengths in
    use on l;

for (all the wavelengths in the network )
   if (Suurballe's algorithm(s, d) succeeds on wavelength λ_i and
   finds two link-disjoint lightpaths p_{i-1} and p_{i-2})
   {
      Store p_{i-1} and p_{i-2};
   }
Scan through all the lightpaths that have been found and return
the pair with the minimum total cost.

//If the previous step fails
for ( all wavelengths in the network )
{
   if ( Dijkstra's algorithm(s, d) on wavelength λ_i succeeds and
   find the minimum cost path p_1 from s to d )
   {
      for ( all λ_j in the network AND λ_j ≠ λ_i )
      {
         remove links on p_1;
         if ( Dijkstra's algorithm(s, d) on λ_j succeeds and find the
         minimum cost path p_2)
            return( p_1 and p_2 ); //Return the lightpaths
         else
            continue;
      }
   }
   else
      continue;
}
return(FAILURE);
```

Fig. 6. Wavelength-Scan Algorithm.

is low, the Route-First Algorithm should have lower blocking probabilities because free wavelengths are readily available and the routes it finds are optimal in total cost. Even though the Wavelength-Scan Algorithm is also likely to find link-disjoint paths under low load, the lightpaths may have longer total length. When the traffic load is high, the Wavelength-Scan Algorithm should have lower blocking probability because it searches through all available wavelengths.

We now modify the two algorithms to support shared protection. We first give a subroutine of finding a shared protection path for a given working path in Fig. 7. With the subroutine, the pseudo code for the Route-First Algorithm with Shared Protection is given in Fig. 8 and the pseudo code for the Wavelength-Scan Algorithm with Shared Protection is given in Fig. 9.

```
find_shared_protection_path(working path p_w)
{
      remove the wavelengths and the network links along p_w;
      remove the wavelengths and the network links along all
other existing working paths;
      remove the wavelength on a link if the number of existing
protection paths traversing the wavelength on that link already
reaches the maximum number that is allowed;
      remove the wavelength on a link if an existing protection
path traverses the wavelength on that link, and the protection
path's corresponding working path and p_w may fail
simultaneously (i.e., they either go through the same link or are
subjected to common risks);
      for the remaining wavelengths and links, set the cost of a
wavelength on a link to zero if an existing protection path
traverse the wavelength on that link;
      for all wavelengths, run Dijkstra's algorithm and choose the
path p_p with minimal cost;
      return(protection path p_p if it is found, or FAILURE if not);
}
```

Fig. 7. Subroutine find_shared_protection_path().

```
initialize c_l to the cost of link l for all fiber links;
for ( all network links)
    increase c_l on link l linearly to the number of wavelengths in
    use on l;

if ( Suurballe's algorithm(s, d) succeed and find two link-disjoint
routes r_1 and r_2 from s to d)
{
   if (find free wavelengths for r_1)
      protection path r_1p = find_shared_protection_path(r_1);

   if (find free wavelengths for r_2)
      protection route r_2p = find_shared_protection_path(r_2);

   compare the total cost of the two path pairs and return the one
   with smaller total cost. Or return FAILURE if no wavelengths
   are available to either path pair;
}
else
   return(FAILURE);
```

Fig. 8. Route-First Algorithm with Shared Protection.

With shared protection, a network can accommodate more connections; hence the blocking probability is reduced. The tradeoffs are complex network management and signaling protocol. For the two algorithms discussed here, calling find_shared_protection_path() costs an additional $O(mn^2 \log n)$ running time, where n is the number of nodes and m is the number of existing paths in the network.

```
initialize c_l to the cost of link l for all fiber links;
for ( all network links)
    increase c_l on link l linearly to the number of wavelengths in
    use on l;

for ( all the wavelengths )
   if (Suurballe's algorithm(s, d) succeeds on wavelength λ_i and
        finds two link-disjoint lightpaths p_{i-1} and p_{i-2})
   {
      protection path p_{i-1p} = find_shared_protection_path(p_{i-1});
      protection path p_{i-2p} = find_shared_protection_path(p_{i-2});
      store the paths temporarily;
   }
Scan through all the lightpaths that have been found and return
the pair with the minimum total cost.
//If the previous step fails
for ( all wavelength λ_i in the network )
{
   run Dijkstra's algorithm(s, d) on λ_i and gets path p_1;
   for ( all λ_j in the network AND λ_j ≠ λ_i )
   {
      protection path p_{1p} = find_shared_protection_path(p_1);
      if (the combined total cost of p_1 and p_{1p} is minimal)
         return(p_1 and p_{1p});
   }
}
return(FAILURE);
```

Fig. 9. Wavelength-Scan Algorithm with Shared Protection.

3. Dynamic lightpath protection under the risk-disjoint constraint

In the previous section, we proved that in a WDM network that has no wavelength conversion capability, the problem of finding a working lightpath and its protection lightpath, each on a different wavelength, is NP-complete. We now consider a WDM network that has wavelength converters at every node.

In such a network, we can use Suurballe's algorithm to find two link-disjoint lightpaths. But if the same risk factor can cause multiple links to fail simultaneously, then a working lightpath and its protection lightpath may still fail simultaneously even if they are link disjoint. Therefore, the working lightpath and protection lightpath must be not only link disjoint but also risk disjoint. Using 3-SAT reduction, [26] proved that this problem is NP-complete if the risks are arbitrarily distributed. It is also possible to use Set-Splitting reduction for the proof. We now give a simpler proof based on the result of the previous problem.

3.1. The Proof of NP-completeness

The problem is formally defined as follows. Given a WDM network with full wavelength conversion $G = (N, L)$, where N is the set of nodes and L is the set of links, and given the SRLGs in G and their distribution, find two risk-disjoint lightpaths from source node s to destination node d, and neither lightpath shares common wavelength with existing lightpaths on the same links (i.e., dedicated protection). We can easily reduce the previous problem to this problem by the procedures given below.

For a network G_0 of two wavelength λ_1 and λ_2, in order to find two link-disjoint lightpaths from node s to d, one on λ_1 and the other on λ_2, we do the following:

1. Construct a network G with the same network nodes as in G_0. For every link in G_0, replace it with two parallel links in G. One link corresponds to wavelength λ_1 and the other corresponds to λ_2. Assign SRLG-1 to all the links corresponding to wavelength λ_1 and assign SRLG-2 to all the links corresponding to wavelength λ_2.
2. For every link in G_0, pick a unique SRLG number and assign it to the two corresponding links in G.

Now every link in G belongs to two SRLGs. One SRLG is associated with a unique link in G_0 and the other (i.e., SRLG-1 or SRLG-2) is associated with the wavelength it represents. If we were able to find two SRLG-disjoint lightpaths from s to d in G, these two paths maps directly to two link-disjoint lightpaths in G_0, one on wavelength λ_1 and the other on wavelength λ_2. Reduction is thus accomplished.

3.2. Proof of NP-completeness for shared protection

When shared protection is allowed, the protection lightpath may share common wavelength with at most T ($T \geqslant 0$) existing protection lightpaths on the same links. We prove the problem to be NP-complete by contradiction. If the problem with shared protection is solvable, then the problem with dedicated protection can also be solved since it is a special case of that with shared protection.

3.3. Risk Distribution in general networks—risk ID and risk set

The concept of SRLG is primarily used by transport network carries to describe the risk sharing by fiber links bundled in a common conduit, or span. We observe that the risk-disjoint constraint is applicable to other connection-oriented networks as well. In order to extend the results of Section 3.1 to general connection-oriented networks, we use the following concepts:

1. Risk ID: For each risk factor that may cause a failure in a network, we assign a unique integer number called the Risk ID. If a network resource, such as a link or a node, is subjected to the risk of one or more failures, then the collection of Risk IDs on that network resource describe all the factors that may cause the resource to fail.
2. Risk Set: A path may traverse multiple network links and nodes. The collection of Risk IDs of the links and nodes is called the Risk Set of the path. The Risk Set represents all the factors that may cause a path to fail. The working path and its protection path must be risk disjoint. In other words, the Risk Sets of the two paths must contain no common Risk IDs.

The concepts of Risk ID and Risk Set are a generalization of SRLG. A single Risk ID represents a SRLG in an optical transport network. In an abstract manner, the concept of Risk ID and Risk Set describes the risks in a network and their associations with network resources, thus it facilitates the implementation of routing algorithms [27] and Integer Linear Program formulations for path protection. If multiple failures are caused by the same risk factor, then all the failures can be represented by a single Risk ID. If we assign an additional unique Risk ID to each network link, then risk-disjoint paths are also link disjoint.

To illustrate these concepts, assume that in a connection-oriented network, such as a WDM wavelength-routed network (Fig. 10), there are seven nodes and eight fiber links. Fiber links *ab* and *ef* cross the same bridge and thus are susceptible to the same risk of a bridge collapse The problem is to find a working path and its protection path from node *s* to node *d*. To solve the problem, we first assign an unique Risk ID to each link. We also assign a Risk ID 9 to both links *ab* and *ef* for the risk of bridge collapse.

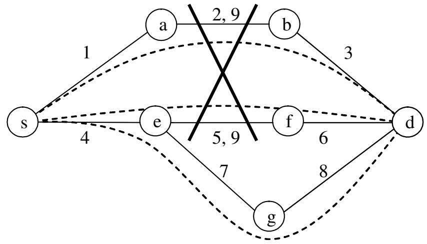

Fig. 10. Example of Risk ID, Risk Set and the risk-disjoint constraint. The numbers indicate Risk IDs of each link.

Now consider path *sabd* and *sefd*. The first path has Risk Set $\{1,2,3,9\}$ and the second path has a Risk Set $\{4,5,6,9\}$. Since the two sets contain common Risk ID 9, they are not risk disjoint and cannot be assigned as the working and protection paths. Next we consider paths sabd and segd. The first path still has Risk Set $\{1,2,3,9\}$ while the second one now has a Risk Set $\{4,7,8\}$, thus they are risk disjoint. Since each physical fiber link has been assigned at least one unique Risk ID, the two paths are also link-disjoint.

With the concept of Risk ID and Risk Set, the problem of dynamic routing of working and protection paths in a general connection-oriented network can be defined as follows. Given network $G = (N, E)$, where N is the set of nodes and E is the set of edges, and given the Risk IDs of each edge, find two risk-disjoint paths from source node s to destination node *d*. The proof in Section 3.1 can be easily generalized to prove that this problem is NP-complete.

3.4. ILP Formulations

In this section, we develop an ILP formulation for the dynamic lightpath protection problem under the risk-disjoint constraint. We also develop two ILP formulations for the problem with additional

SLA and priority constraints. Since the traffic is dynamic, the ILP formulations should be solved for each incoming connection request.

3.4.1. ILP formulation for the problem under only the risk disjoint constraint

The first ILP formulation is for the connection requests under only the risk-disjoint constraint. The following are given as inputs to the problem:

- N: number of nodes in the network.
- L: collection of all fiber links in the network.
- W_{ij}: number of free wavelengths on link $ij \in L$; W_{ij} takes on the value of 0 if all wavelengths are taken by previously established lightpaths.
- $C = \{c_1, c_2, \ldots, c_k, \ldots, c_T\}$: collection of all Risk IDs in the network. T is the number of Risk IDs in the network.
- r_{ij}^k: 1 if link $ij \in L$ has Risk ID c_k; 0 otherwise.
- s: source node.
- d: destination node.

The ILP solves the following variables:

- α_{ij}^{sdw}: 1 if wavelength w on link ij is taken by the working lightpath from source s to destination d; 0 otherwise.
- β_{ij}^{sdw}: 1 if wavelength w on link ij is taken by the protection lightpath from source s to destination d; 0 otherwise.

Objective: Find a working lightpath and a protection lightpath that satisfy the risk-disjoint constraint.

$$\sum_{\forall ij \in L} \sum_{w}^{W_{ij}} \alpha_{ij}^{sdw} + \sum_{\forall ij \in L} \sum_{w}^{W_{ij}} \beta_{ij}^{sdw} > 0 \tag{3.1}$$

Constraints:
Link-disjoint constraint by the problem definition:

$$\sum_{w}^{W_{ij}} \alpha_{ij}^{sdw} + \sum_{w}^{W_{ij}} \beta_{ij}^{sdw} \leqslant 1, \quad 1 \leqslant i,j \leqslant N \tag{3.2}$$

Risk-disjoint constraint:

$$r_{ij}^k \sum_{w}^{W_{ij}} \alpha_{ij}^{sdw} + r_{mn}^k \sum_{w}^{W_{mn}} \beta_{mn}^{sdw} \leqslant 1 \quad \forall k \leqslant T, \quad \forall ij \in L, \quad \forall mn \in L \tag{3.3}$$

Flow-conservation without the wavelength continuity constraint:

$$\sum_{i=1}^{N} \sum_{w}^{W_{il}} \alpha_{il}^{sdw} - \sum_{j=1}^{N} \sum_{w}^{W_{lj}} \alpha_{lj}^{sdw} = \begin{cases} 1, & \text{if } l = d \\ -1, & \text{if } l = s \\ 0, & \text{otherwise} \end{cases} \quad 1 \leqslant l \leqslant N \tag{3.4}$$

$$\sum_{i=1}^{N} \sum_{w}^{W_{il}} \beta_{il}^{sdw} - \sum_{j=1}^{N} \sum_{w}^{W_{lj}} \beta_{lj}^{sdw} = \begin{cases} 1, & \text{if } l = d \\ -1, & \text{if } l = s \\ 0, & \text{otherwise} \end{cases} \quad 1 \leqslant l \leqslant N \tag{3.5}$$

3.4.2. ILP formulation for the problem under additional SLA and high priority constraints

The second ILP formulation is for the connection requests with a high priority that requires strict risk disjointness, and an SLA constraint on the maximum delay on both the working lightpath and the protection lightpath. The following are given as inputs to the problem:

- All the inputs in the first ILP formulation.
- $\langle d_{ij} \rangle$: Propagation delay on fiber link ij, $\forall ij \in L$
- D: The maximum delay for both lightpaths.
- The ILP solves the following variables:
- All the variable to be solved in the first ILP formulation.

Objective: Same as Eq. (3.1).
Constraints:
Constraints (3.2), (3.3), (3.4), (3.5)

$$\sum_{\forall ij \in L} \sum_{\forall w \in \Lambda_{ij}} d_{ij} \alpha_{ij}^{sdw} \leqslant D \tag{3.6}$$

$$\sum_{\forall ij \in L} \sum_{\forall w \in \Lambda_{ij}} d_{ij} \beta_{ij}^{sdw} \leqslant D \tag{3.7}$$

3.4.3. ILP formulation for the problem under additional SLA and low priority constraints

The third ILP formulation is for the connection requests with the additional SLA constraint on the maximum delay, and a low priority that allows risk sharing by the working lightpath and the protection lightpath. The following are given as inputs to the problem:

- All the inputs in the second ILP formulation. The ILP solves the following variables:
- All the variable to be solved in the first ILP formulation.
- δ_1^k: 1 if Risk ID c_k is on the working lightpath; 0 otherwise.
- δ_2^k: 1 if Risk ID c_k is on the protection lightpath; 0 otherwise.
- $overlap^k$: 1 if Risk ID c_k is on both the working lightpath and the protection lightpath; 0 otherwise.

Objective: Find a working lightpath and a protection lightpath that share the minimum number of risks.

$$\text{Minimize} \quad \sum_{\forall ij \in L} \text{overlap}^k \tag{3.8}$$

Constraints:
Constraints (3.2), (3.4), (3.5), (3.6), (3.7)

$$\delta_1^k \leqslant \sum_{ij} \sum_{w}^{W_{ij}} (r_{ij}^k \cdot \alpha_{ij}^{sdw}) \tag{3.9}$$

$$|L| \cdot \delta_1^k \geqslant \sum_{ij} \sum_{w}^{W_{ij}} (r_{ij}^k \cdot \alpha_{ij}^{sdw}) \tag{3.10}$$

$$\delta_2^k \leqslant \sum_{ij} \sum_{w}^{W_{ij}} (r_{ij}^k \cdot \beta_{ij}^{sdw}) \tag{3.11}$$

$$|L| \cdot \delta_2^k \geqslant \sum_{ij} \sum_{w}^{W_{ij}} (r_{ij}^k \cdot \beta_{ij}^{sdw}) \tag{3.12}$$

$$0 \leqslant \delta_1^k + \delta_2^k - 2 \cdot \text{overlap}^k \leqslant 1 \tag{3.13}$$

3.5. Heuristic algorithms

To solve the dynamic lightpath protection problem under the risk-disjoint constraint, [27] and [28] proposed a simple two-step algorithm as mentioned in Section 1. This algorithm first increases the cost of those links whose Risk ID appears more than once in the network. The higher the link cost, the less likely that link will be chosen by the first minimum cost path; hence when routing the second minimum cost path, there are more links available that are risk disjoint from the first minimum cost path.

The simple two-step algorithm fails in the "trap" topologies because the first minimum cost path is obtained without considering the disjoint path being routed next. Suurballe's algorithm overcomes the problem by jointly routing both paths and minimizing the total cost. We combine these two algorithms and develop the Joint-Search Two-Step Algorithm. As the first step of this algorithm, we increase the cost of those fiber links whose Risk ID appears more than once in the network. In order to balance traffic load among the network links, we also adjust the cost of a link based on the amount of existing traffic carried on that link. Next we run Suurballe's algorithm and obtain two link disjoint lightpaths from the source to the destination. Note that these two paths may not be risk disjoint. We select these two paths one at a time, and designate it as the working lightpath and find its corresponding protection lightpath using the find_protection_path() procedure in Fig. 11. From the resulting two pairs of risk-disjoint paths, we choose the pair with smaller total cost. The pseudo code for the Joint-Search Two-Step Algorithm is given in Fig. 12. Please note that the pseudo codes only serve to illustrate the basic ideas. All the modifications made to the network topology are temporary and stay only within the scope of the local procedures.

Both the simple two-step algorithm and the Joint-Search Two-Step Algorithm are adaptive to

```
find_protection_path(working path p_w)
{
    remove the network links along p_w;
    remove the network links used by other working paths;
    remove the links that have common Risk IDs with p_w;
    run Dijkstra's algorithm(s, d). If Succeeds, return path p_p ;
    Otherwise return FAILURE;
}
```

Fig. 11. Subroutine find_protection_path().

```
initialize c_l to the cost of link l for all fiber links;
for (all network links)
{
   for (all Risk ID R_l that occurs more than once in the network)
   {
       increase link cost c_l on link l linearly to (n_{R_l} – 1) if l
       contains Risk ID R_l, where n_{R_l} is the number of R_l's
       occurrences in the network;
   }
   increase c_l on link l linearly to the number of wavelengths in
   use on l;
}

if ( Suurballe's algorithm(s, d) finds paths p_1 and p_2)
{
   protection path p_1p = find_protection_path(p_1);
   protection path p_2p = find_protection_path(p_2);

  compare the total cost of the two path pairs, i.e., (p_1, p_1p) and
  (p_2, p_2p), and choose the pair with smaller total cost. If succeed,
   return the two paths. Otherwise return FAILURE;
}
return(FAILURE);
```

Fig. 12. Joint-Search Two-Step Algorithm.

the dynamic network status. Yet the Joint-Search Two-Step Algorithm is superior because it may find two disjoint paths in networks where the simple two-step algorithm fails. Compared to other heuristics designed specifically for fiber span or duct-layer constraint, the Joint-Search Two-Step Algorithm works on networks with arbitrary risk distribution, including configurations where a fiber link belongs to multiple spans, and thus has more than one Risk ID. If every Risk ID occurs only once in the network, this algorithm is equivalent to Suurballe's algorithm. It also has the same order of time complexity as Suurballe's algorithm.

We can further improve the performance of the Joint-Search Two-Step Algorithm with shared protection. When shared protection is supported, a network can accommodate more connections thus the blocking probability is reduced. We first define a subroutine of finding a shared protection path for a given working path (Fig. 13). The pseudo code for the Joint-Search Two-Step Algorithm with Shared Protection is given in Fig. 14. The most time-consuming portion of this algorithm is calling find_shared_protection_path(). This algorithm runs in $O(mn^3 \log n)$ time, where n is the number of nodes and m is the number of existing paths in the network.

```
find_shared_protection_path(working path p_w)
{
      remove all network links along p_w;
      remove network links along all other existing working
paths;
      remove a link if the number of existing protection paths
traversing that link already reaches the maximum number that is
allowed;
      remove a link if an existing protection path traverses that
link, and the protection path's corresponding working path and
p_w have a common Risk ID in their Risk Set;
      for the remaining links, set the cost of a link to zero if an
existing protection path traverses that link;
      run Dijkstra's algorithm and choose the path p_p with
minimal cost;
      return(protection path p_p if it is found, or FAILURE if not);
}
```

Fig. 13. Subroutine find_shared_protection_path().

```
initialize c_l to the cost of link l for all fiber links;
for (all network links)
{
   for (all Risk ID R_l that occurs more than once in the network)
   {
       Increase link cost c_l on link l linearly to (n_{R_l} – 1) if l
       contains Risk ID R_l, where n_{R_l} is the number of R_l's
       occurrences in the network;
   }
   increase c_l on link l linearly to the number of wavelengths in
   use on l;
}

if ( Suurballe's algorithm(s, d) finds paths p_1 and p_2)
{
   p_1p = find_shared_protection_path(p_1);
   p_2p = find_shared_protection_path(p_2);

  compare the total cost of the two path pairs, i.e., (p_1, p_1p) and
  (p_2, p_2p), and choose the pair with smaller total cost. If
  succeed, return the two paths. Otherwise return FAILURE;
}
else
   return(FAILURE);
```

Fig. 14. Joint-Search Two-Step Algorithm with Shared Protection.

4. Simulations

We have developed ILP formulations and heuristic algorithms for two NP-complete dynamic lightpath protection problems. For the problem under the wavelength continuity constraint, we developed the Route-First Algorithm and the

Wavelength-Scan Algorithm with dedicated protection. We also developed shared protection support for both of them. For the problem under the risk disjoint constraint, we developed the Joint-Search Two-Step Algorithm with dedicated protection and with shared protection. Computer simulations were conducted to evaluate the performance of these algorithms. Since the problems are considered under dynamic traffic, the primary performance metric is the blocking probability. We use various network topologies in the simulations and obtain the results with confidence level between 90% and 95%, and confident interval around 5%.

4.1. Simulations of the route-first algorithm and the wavelength-scan algorithm

Recall that the Route-First Algorithms first find two minimum-cost link-disjoint routes and then assign free wavelengths to them. The Wavelength-Scan Algorithms search through all available free wavelengths for a pair of link-disjoint lightpaths. We compare the two algorithms with the fixed alternate paths heuristic proposed in [22]. As introduced in Section 1, the fixed alternate paths heuristic predefines two groups of alternate routes for each *s–d* pair, i.e., one working group of *M* routes and one protection group of *B* routes.

We first use a randomly generated 40-node network topology with an average nodal degree of 4. The cost of every link is assumed to be 1, and the number of wavelengths on each link is 8. A lightpath requires one wavelength on every link it traverses. We also assume connection requests arrive according to a Poisson process, and holding times are exponentially distributed.

For the fixed alternate paths heuristic in this network topology, the maximum average number of alternate paths in the working group and in the protection group is 4. Therefore we run the simulations three iterations with $M = B = 2, 3, 4$, respectively. We also run simulations for the Route-First Algorithm and the Wavelength-Scan Algorithm. The simulations are performed under various traffic loads. The blocking probabilities are depicted in Fig. 15. It is clear that both the Route-First Algorithm and the Wavelength-Scan Algorithm have lower blocking probabilities.

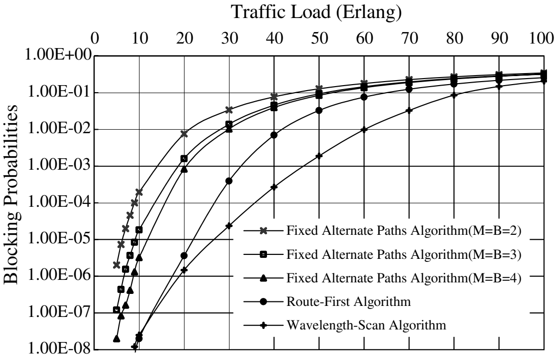

Fig. 15. Blocking probability versus load for the Fixed Alternate Paths Algorithms, the Route-First Algorithm and the Wavelength-Scan Algorithm.

We can also use the 16-node, 25-link NSFNET backbone topology (Fig. 16) [33,34] for the simulation. The assumptions for this topology are the same as those in the 40-node topology. The average nodal degree of this network is 2.875. For the fixed alternate paths heuristic in this topology, the maximum average number of alternate paths in the working group and in the protection group is 3. The simulation results are depicted in Fig. 17.

From both simulations we observe that the two heuristics we develop may achieve significantly lower blocking probabilities than the fix alternative paths heuristic. Using other network topologies yield similar results. As we discussed in Sections 1 and 2, the Route-First Algorithm and the Wavelength-Scan Algorithm compute the working lightpath and the protection lightpath

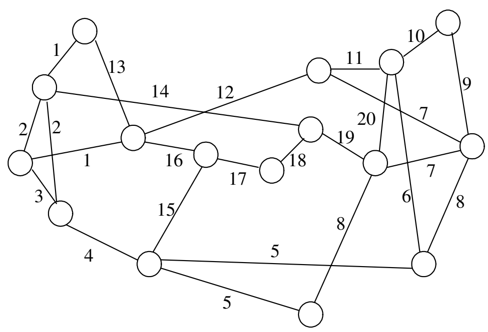

Fig. 16. 16-node NSFNET backbone network. The numbers indicate Risk IDs.

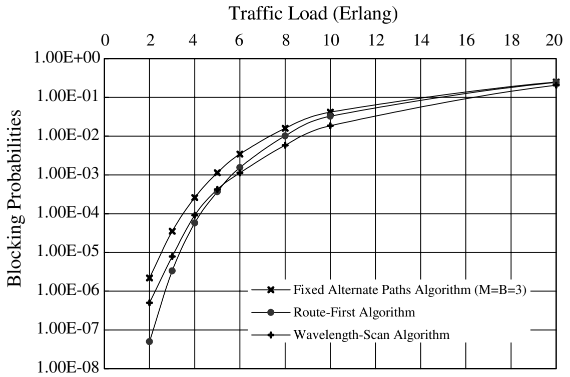

Fig. 17. Blocking probability versus load for the Fixed Alternate Paths Algorithm, the Route-First Algorithm and the Wavelength-Scan Algorithm. For traffic load above 20 Erlangs, the blocking probabilities of the three algorithms become indistinguishable.

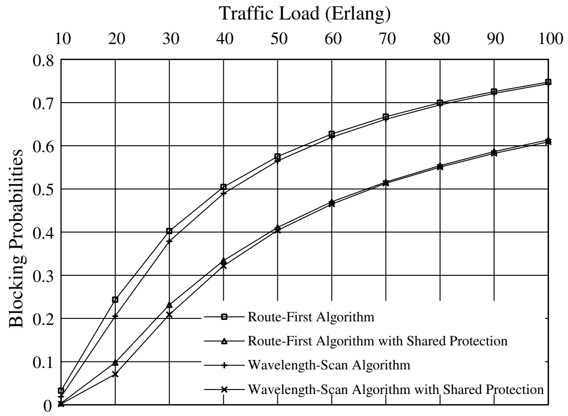

Fig. 19. Blocking probability versus load for the Route-First algorithms and the Wavelength-Scan algorithms under high traffic loads.

for each incoming connection request based on real-time network status while fixed alternate paths heuristic use predefined routes. Therefore the Route-First Algorithm and the Wavelength-Scan Algorithm are more adaptive than the fixed alternate paths heuristic.

Next, we compare the blocking probabilities of the Route-First Algorithms and the Wavelength-Scan Algorithms with each other, including the ones with shared protection. The results are depicted in Figs. 18 and 19.

From this simulation, we first observe that shared protection significantly improves blocking

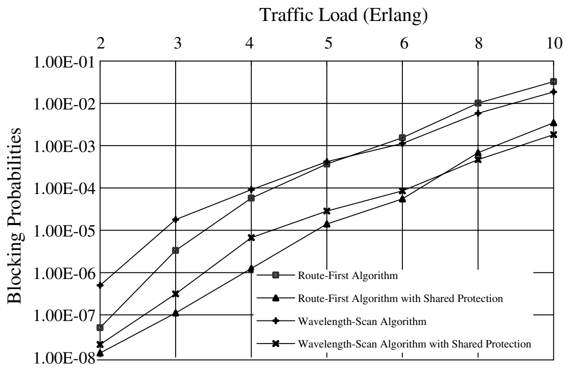

Fig. 18. Blocking probability versus load for the Route-First algorithms and the Wavelength-Scan algorithms under low traffic loads.

probability, regardless of the traffic load. Secondly, when the traffic load is very low, the Route-First Algorithms perform better than the Wavelength-Scan Algorithms. While the traffic load increases, the Wavelength-Scan Algorithms become better than the Route-First Algorithms. As we explained in Section 2, when the traffic load is low, free wavelengths are readily available, and the lightpaths obtained from the Route-First Algorithms are minimal in total length. Even though the Wavelength-Scan Algorithms are also very likely to find link-disjoint paths under low load, the lightpaths may have longer total length. Therefore, the Route-First Algorithms have lower blocking probabilities. When the traffic load becomes higher, the Wavelength-Scan Algorithms have lower blocking probability because they perform more thorough search through all available wavelengths. The simulation results match our expectation.

4.2. Simulations of the simple two-step algorithm and the joint-search two-step algorithm

In this simulation, we evaluate the Joint-Search Two-Step Algorithm for the dynamic path protection problem under the risk-disjoint constraint. In comparison, we also run computer simulations on the Simple Two-Step Algorithm [27,28] and compare their blocking probabilities.

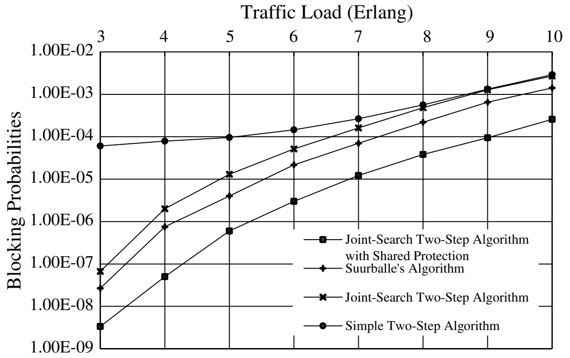

Fig. 20. Blocking probability versus load under low traffic load.

We use the same 16-node NFSNET backbone topology as the one used in the previous simulation, but Risk IDs are randomly assigned to the links. Assumptions about link cost, wavelengths, connection requests arrivals and departures stay the same. In addition, we assume full wavelength conversion at every node. Other network topologies are also used and yield similar results.

Since an optimal solution is infeasible due to the NP-completeness of the problem, we run Suurballe's algorithm without the risk-disjoint constraint and use the resulting blocking probabilities as a lower bound for the other two heuristics with dedicated protection. Note that the disjoint paths obtained from Suurballe's algorithm may not be risk disjoint. The simulation results are depicted in Figs. 20 and 21.

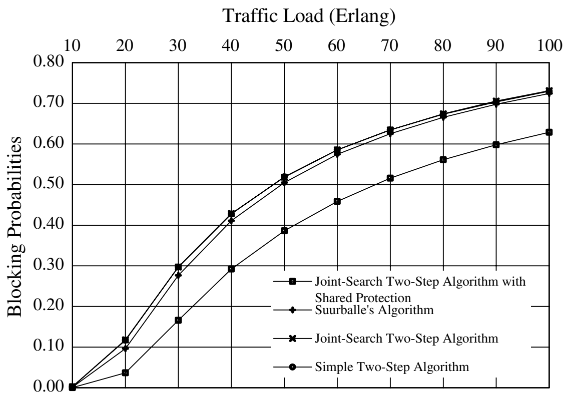

Fig. 21. Blocking probability versus load under high traffic load.

The simulation results illustrate that, first of all, shared protection significantly improves the blocking probabilities, regardless of the traffic load. Secondly, when the traffic load is low, the Joint-Search Two-Step Algorithm is significantly better than the simple two-step algorithm. As the traffic load increases, the blocking probabilities of the two algorithms converge. The performance improvement of the Joint-Search Two-Step algorithm stems from its incorporation of Suurballe's algorithm and the minimization of the total length of the working path and its protection path. Thus this algorithm is an effective solution for finding risk-disjoint working path and its protection path.

5. Conclusion

In this paper we considered two fundamental problems on dynamic lightpath protection in WDM mesh networks. In the first problem, all lightpaths are subject to the wavelength continuity constraint. The objective is to find link-disjoint working lightpath and protection lightpaths, each on a different wavelength. In the second problem, wavelength conversion eliminates the wavelength continuity constraint but a single risk factor may cause multiple links to fail simultaneously. The objective is to find link-disjoint working lightpath and protection lightpaths that are also risk disjoint. We proved that the two problems are NP-complete. The second problem can be generalized to any connection-oriented networks with the introduction of two new concepts—*Risk ID* and *Risk Set*.

To solve these two NP-complete problems, we developed ILP formulations and heuristic algorithms. We conducted computer simulations to evaluate their blocking probabilities under various traffic loads. The simulations confirm that shared protection significantly improves blocking probability over dedicated protection. The simulation also reveals that, for the first problem, the two heuristics we developed archive significantly lower blocking probability than the previously proposed fixed alternate paths heuristic. When compared to each other, the Route-First Algorithm performs better than the Wavelength-Scan Algorithm under low traffic load. The Wavelength-Scan Algorithm performs better than the Route-First Algorithm under high traffic load. On the second problem, the Joint-Search Two-Step Algorithm is superior to the simple two-step algorithm.

One possible area of future work may be to further improve the performance of the Wavelength-Scan Algorithm at higher load. In addition to traffic balancing, we may also adjust the link costs based on other factors such as the number of free wavelengths on a link that are reachable to the destination. This type of adjustments may improve the algorithm's performance. Similarly, we may further improve the performance of the Joint-Search Two-Step Algorithm. The algorithm currently adjusts the link costs based on the occurrences the Risk IDs of the links in the network. This adjustment has a significant impact on the algorithm's performance. We can add other factors to make the adjustment more intelligent.

Appendix A. Suurballe's algorithm

Suurballe's algorithm and its variations find a pair of link-disjoint paths from a source node s to a destination node d as long as such paths exist in a network. The total cost of the resulting two link-disjoint paths is minimal among all such path pairs. The algorithm runs in $O(n^2 \log n)$ time, where n is the number of nodes. We use the network topology in Fig. 1 to illustrate the algorithm (Fig. 22). More information on this subject can be found in [18,19].

Appendix B. 3-Satisfiability Problem

The 3SAT problem is a well-known NP-complete problem. The problem is stated as follows. Given a collection of clauses $C = \{C_1, C_2, \ldots, C_M\}$ on a finite set of variables $V = \{v_1, v_2, \ldots, v_N\}$ such that $|C_j| = 3$ for $1 \leqslant j \leqslant M$, where clause C_j is the boolean "or" of three literals (a literal is either a variable or the boolean "not" of a variable) and is satisfied by a truth assignment if and only if at least one of the three literals is true, is there a truth assignment for V that satisfies all the clauses in C?

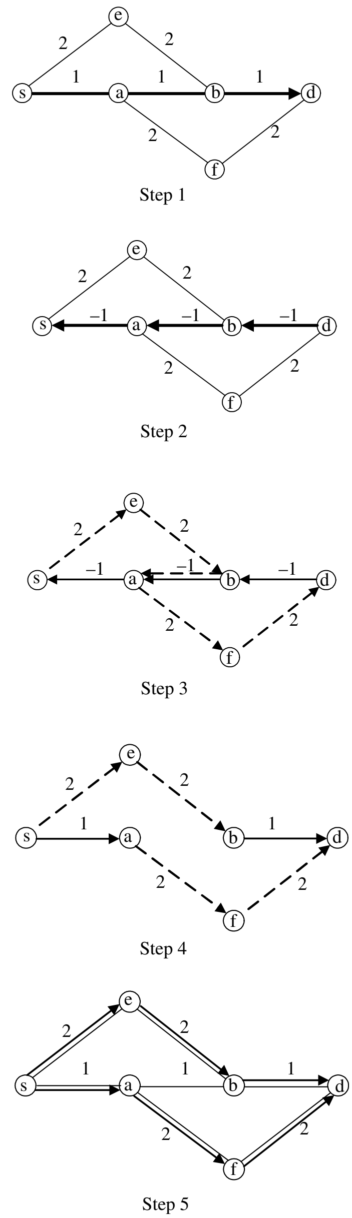

Fig. 22. Illustration of Suurballe's algorithm. Step 1. Find the minimum cost path p_1 (*s–a–b–d*). Step 2. Make the links along p_1 unidirectional pointing to the source node *s*. Step 3. Run shortest path algorithm again and find another minimum cost path p_2 (*s–e–b–a–f–d*). Step 4. Remove the overlapped links on p_1 and p_2. Step 5. Combine the remaining links on p_1 and p_2 and get two link-disjoint paths *s–e–b–d* and *s–a–f–d*.

As an example given in the paper, the set of all variables is $V = \{v_1, v_2, v_3\}$, the clauses are $C =$

$\{C_1, C_2\}$ where $C_1 = v_1 \vee \bar{v}_2 \vee v_3$, $C_2 = \bar{v}_1 \vee v_2 \vee v_3$. One of the truth assignments that makes both C_1 and C_2 true is $v_1 = 0$, $v_2 = 0$, $v_3 = 1$. Another truth assignments is $v_1 = 1$, $v_2 = 1$, $v_3 = 1$.

The 3-SAT problem is one of the earliest problems found to be NP-complete. It is used as a base problem for reduction to prove the NP-completeness of many other problems. The principle of the reduction is as follows. If a known NP-complete problem P_1 can be transformed to another problem P_2 with polynomial complexity, then P_2 must be NP-complete, otherwise P_1 becomes solvable with zpolynomial complexity.

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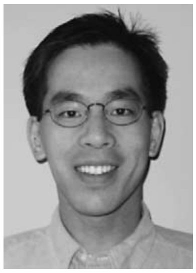

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