Reliable lightpath routing in optical mesh networks under multiple link failures¹

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Abstract - In this work, we study the problem of maximizing lightpath reliability in optical mesh networks against simultaneous failures of multiple fiber links without using protection schemes. The fiber links belong to shared risk link groups (SRLGs) that have arbitrary failure probabilities. This problem is NP-hard and we propose heuristic algorithms for networks with large numbers of SRLGs as well as optimal solutions for networks with smaller numbers of SRLGs. The solutions are evaluated through simulations.

Keywords – lightpath reliability; multiple failures; Shared Risk Link Group (SRLG)

I. INTRODUCTION

A lightpath in wavelength-division-multiplexing (WDM) networks offers data rate up to 40 Gbits/s or even 100Gbits/s, which makes it an ideal transmission medium for the next generation transport networks [1][2][3][4]. With such high data rates, a failure of a lightpath can potentially cause significant service disruption to the customers. Therefore it is among the top priorities of network operators to ensure lightpath reliability.

A common approach to achieve high reliability is through lightpath protection in which a link-disjoint protection lightpath is pre-computed and reserved for each working lightpath [5][6]. Such schemes provide 100% reliability against single-link failures. However, various risk factors such as natural and man-caused catastrophes introduce the possibility that when a network failure occurs, multiple fiber links that belong to the same shared risk link group (SRLG) fail simultaneously [7][8][9][10]. In this case, only a working and a protection lightpath that are SRLGdisjoint can survive the failure [11]. However, the study in [12] proved it NP-hard to find SRLG-disjoint working and protection lightpaths. Consequently, attempts were made to minimize the probability of simultaneous failures of the pair. Yuan et al. proved this problem also NP-hard and proposed heuristic algorithms for the special cases in which all SRLGs have equal failure probabilities [13][14]. Various heuristic solutions were also proposed for the general case in which the SRLGs have different failure probabilities [15][16][17].

In addition to the difficulties of finding the maximum reliable working and protection lightpaths, another limitation of the protection schemes is that they require the reservation of a significant amount of network resources. Shared path protection may be used to improve resource utilization at the expense of signaling and network management [17][18]. Therefore for certain types of traffic and customers, it may be more resource-efficient and costeffective to use a single lightpath without protection, if the reliability of the lightpath can be maximized.

As later shown in Section II, for networks with only single failures, finding the maximum reliable lightpath without protection is equivalent to a minimum-cost-path problem and can be easily solved. However, simultaneous multiple failures complicate the matter and make the task NP-hard [13][14]. The work in [13][14] proposed heuristic solutions for the special cases in which all SRLGs have equal failure probabilities. In this paper, we study the maximum reliable lightpath problem for the general case in which the SRLGs have arbitrary failure probabilities. We develop both heuristic and optimal solutions. A search of recent literatures has indicated this work is the first to study the general problem without using protection schemes.

The remainder of the paper is organized as follows. In Section II, we describe the maximum reliable lightpath problem. In Section III, we propose optimal solutions for networks with small number of SRLGs. We also propose heuristic solutions for networks with large number of SRLGs. The algorithms are evaluated through computer simulations in Section IV. We conclude the paper in Section V.

II. PROBLEM DESCRIPTION

The problem is defined as follows. Given network $G = (N, L, S, P_s)$ where N is the set of nodes, L is the set of fiber links (assume they are bidirectional), and S is the set of SRLGs in the network, $P_s = \{p_l, p_2, p_3, ..., p_i, ...\}$ is the set of non-failure probabilities of each SRLG $s_i \in S$ and $0 < p_i < 1$, also given $S^l \forall l \in L$ and $S^l \subseteq S$ is the set of SRLGs to which link l belongs, find one lightpath P from source node s to destination node d such that P has the maximum reliability.

The reliability of a lightpath is defined as follows. Let S^{P} be the set of all SRLGs to which the links of a lightpath P belong. Then the reliability of P is

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$$r^{P} = \prod_{s_{i}^{P} \in S^{P}} p_{i}, p_{i} \in P_{s} \text{ and } p_{i} \text{ is the non-failure}$$

probability of SRLG s_{i}^{P} (1)

In this study, we focus on fiber link failures and not node failures for the following reasons. First of all, nodal devices in modern transport networks often have built-in redundancies and are located in well-controlled facilities, which make them very reliable; and secondly, the failure of a node can be equaled to the simultaneous failures of all the links ending with that node [19].

A single SRLG may contain multiple fiber links while a fiber link may belong to multiple SRLGs. However, we can transform a link belonging to multiple SRLGs into multiple links each belonging to a single SRLG without altering the reliabilities of the lightpaths in the network. This is explained as follows.



a. Link *l* between nodes n_1 and n_2 belonging to multiple SRLGs: S_1^l ,



b. Multiple concatenating links each belonging to a single SRLGs: l_i to s_1^l , l_2 to s_2^l , ..., l_i to s_i^l ,...

Figure 1. Transforming a fiber link that belongs to multiple SRLGs into multiple links each belonging to a single SRLG.

Let the set of SRLGs to which *l* belongs be $S^{l} = \{s_{1}^{l}, s_{2}^{l}, \dots, s_{i}^{l}, \dots\}$ and $|S^{l}| > 1$. The non-failure probability of *l* is

$$r^{l} = \prod_{s_{i}^{l} \in S^{l}} p_{i}$$
, $p_{i} \in P_{s}$, and p_{i} is the non-failure

probability of SRLG
$$s_i^l$$
 (2)

Now replace l with $|S^l|$ concatenating links $l_1, l_2, ..., l_i$, ..., each belongs to a unique SRLG $s_1^l, s_2^l, ..., s_i^l, ...$ from S^l , as shown in Fig. 1. The reliability of l_i is the non-failure probability of s_i^l , i.e., p_i . Thus the reliability of the combined links $l_1, l_2, ..., l_i, ...$ is $p_1 \times p_2 \times ... \times p_i \times ...$, which is the same as r^l . Also because there is no other links branching off from these links, a lightpath going through one of the links must also go through all of them. Therefore their effects on the reliabilities of lightpaths are the same as that of l. Consequently, without the loss of generality, we assume for the remainder of the paper, all fiber links each belongs to a single SRLG, i.e., $|S'| = 1 \forall l \in L$.

III. ANALYSIS AND SOLUTIONS

In the special case in which every fiber link belongs to a unique SRLG, the maximum reliable lightpath can be readily obtained using the following algorithm:

Algorithm (A-1)	
Step 1:	for (every link $l_i \in L$) {
-	set its link cost to
	$c_i = log p_i $ where p_i is the non-failure
	probability of SRLG s_i^l to which l_i
	belongs;
	}
Step 2:	run Dijkstra's algorithm for a minimum-cost
	path between node s and d; [20]
Step 3:	if (Step 2 succeeds)
-	return the path obtained in Step 2;

This algorithm's running time is the time of computing the cost of every link plus the minimum-cost-path algorithm, i.e., $O(|L|+|N|\log|N|)$ [20]. Its correctness can be proven as follows.

Proof: Let *P* be a lightpath connection node *s* and *d*. Let the collection of links of *P* be $L^P = \{l_1^P, l_2^P, ..., l_i^P, ...\}$. From Step 1, the link cost c_i of link l_i^P is $|log p_i|$ where p_i is the non-failure probability of the SRLG s_i^l to which l_i^P belongs, and $0 < p_i < 1$.

Thus the total cost of lightpath P is

$$c^{P} = \sum_{l_{i}^{P} \in L^{P}} c_{i} = \sum_{l_{i}^{P} \in L^{P}} |log p_{i}| = |log(\prod_{l_{i}^{P} \in L^{P}} p_{i})|$$

= $log(\prod_{l_{i}^{P} \in L^{P}} p_{i})^{-1}$, and $1 < (\prod_{l_{i}^{P} \in L^{P}} p_{i})^{-1} < +\infty$

Hence, $\prod_{l_i^P \in L^P} p_i$ is maximized when c^P is minimized.

Since every link belongs to a unique SRLG, the reliability of *P* is $r^P = \prod_{l_i^P \in L^P} p_i$, $0 < r^P < 1$. Thus when c^P is

minimized, r^{P} is maximized.

This problem becomes NP-hard if each SRLG contains multiple fiber links that may fail simultaneously [13][14]. However, for networks in which the number of SRLGs is small, we propose two optimal solutions whose running times are polynomial to the number of network nodes and are exponential only to the number of SRLGs. For networks with arbitrarily large number of SRLGs, we propose two heuristic solutions whose running times are polynomial to both the number of network nodes and that of SRLGs.

For both of the optimal algorithms, we need to generate

all the subsets from a set of SRLGs. This is equivalent to the problem of generating combinations, which has been extensively studied [21][22]. Thus we omit the detail of the procedure. The first optimal solution (i.e., OA-1) is described as follows:

Algorithm (OA-1)

Step 1: same as Step 1 of (A-1); Step 2: generate all non-empty subsets of S and store them in array SUB[]; for (every subset S^{x} in SUB[]) { compute $r^{x} = \prod_{s_{i} \in S^{x}} p_{i}$, where p_{i} is the nonfailure probability of SRLG s_i; } sort SUB[] in the descending order of the r^x values of each subset S^{x} in SUB[]; for (each subset S^{x} in SUB[], starting from Step 3: SUB[0]) { line^{*}: remove all links in G whose SRLGs $\notin S^{x}$; run Dijkstra's Algorithm for a minimum-cost path P^x between s and d; restore the links that were removed in line^{*}; if (succeed in obtaining P^{x}) { return P^x and quit; output "No path exist" and quit;

The total number of non-empty subsets generated in Step 2 is $2^{|S|}$ -1. Thus the running time of Step 2 is $O((2^{|S|}-1)log(2^{|S|}-1)) = O(|S|2^{|S|})$. The running time of Step 3 is $O(2^{|S|}|N|log|N|)$. Thus the total running time of (OA-1) is

$$RT_1 = O(2^{|S|}(|S| + |N| \log |N|))$$
(3)

This running time is polynomial to the number of network nodes |N| and is exponential only to |S|, i.e., the number of SRLGs. Therefore (OA-1) is a viable solution for networks in which the value of |S| is small.

Proof of correctness:

If a path does not exist between s and d, Step 3 cannot generate a path from any of the subnetworks of G which it iterates through. Thus (OA-1) correctly produces no lightpath.

If there does exist an optimal path P_{θ} from *s* to *d* with the maximum reliability r_0^{P} and non-empty SRLG set S_0^{P} . We need to prove (OA-1) produces a lightpath of reliability r_0^{P} .

Because *SUB[]* has all non-empty subsets of *S* including S_0^P , sorted by their r^x values in descending order, Step 3 cannot terminate in an iteration with a subset S^x and an r^x value that is greater than r_0^P , otherwise it would indicate that there exists a path P_1 from *s* to *d* in the subnetwork that has all the SRLGs in S^x . From (1), P_1 would have reliability

no less than r^x thus greater than r_0^P .

If P_0 does exist, Step 3 cannot go through all iterations without generating a path, nor terminate in an iteration on a subset S_i^P with $r_i^P < r_0^P$. This is because for every SRLG subset S_i^P with $r_i^P < r_0^P$, the index of S_i^P in *SUB[]* must be greater than that of S_0^P . If Step 3 were to reach an iteration on S_i^P , it must have executed an earlier iteration on S_0^P in which the algorithm should have already terminated because of the existence of P_0 between *s* to *d*.

Therefore, this algorithm can only terminate in an iteration on an SRLG subset S^x with $r^x = r_0^P$, which generates a maximum reliable lightpath, if such lightpath does exist.

To improve upon (OA-1), we note that all lightpaths connecting *s* and *d* must go through fiber links that end with *s* and *d*. Let S^s be the set of SRLGs to which the links ending with *s* belong. Let S^d be the set of SRLGs to which the links ending with *d* belong. S^s and S^d may contain common SRLGs. If a lightpath does not consist of fiber links that belong to SRLGs in S^s and S^d , the lightpath cannot connect *s* and *d*. Thus we can exclude these lightpaths from consideration. Hereby we propose the second optimal solution (OA-2) with details as follows.

Algorithm (OA-2) same as Step 1 of (A-1); Step 1: Step 2: r' = 0: P' = null;for (every subset S^{x} in $S - S^{s} - S^{d}$) { Step 3: for (every non-empty subset S^{v} in $S^{s} \cap S^{d}$) { Step 3.1: for (every subset S^{ϵ} in S^{ϵ} - $S^{\epsilon} \bigcap S^{d}$) { line[†]: remove all links whose SRLGs ∉ $(S^x \cup S^y \cup S^z);$ run Dijkstra's Algorithm for a minimum-cost path P_{xyz} between s and d; restore the links that were removed in line[†]: *if* (succeed in obtaining P_{xyz}) { compute the reliability of the path r_{rnz}^{P} using (1); $if(r_{xvz}^{P} > r')$ $r' = r_{xvz}^P;$ $P' = P_{xvz}$; line[‡]: } } for (every non-empty subset S^{v} in S^{d} - $S^{s} \cap S^{d}$) Step 3.2: for (every non-empty subset S^{z} in S^{s}) { same as the lines between line^{\dagger} and

	line [‡] ;
	}
	}
	}
Step 4:	return P';

Proof of correctness:

All SRLGs in *G* can be grouped into four nonoverlapping sets: $S^s - S^s \cap S^d$, $S^d - S^s \cap S^d$, $S^s \cap S^d$ and $S - S^s - S^d$. Let P_0 be the optimal lightpath from *s* to *d* with the maximum reliability r_0^P and SRLG set S_0 . Since S_0 must contain SRLG(s) in S^s and S^d , one of following three conditions must be true:

a. S_0 contains a subset of $S - S^s - S^d$, a non-empty subset of $S^s \cap S^d$, a subset $S^s - S^s \cap S^d$ and nothing in $S^d - S^s \cap S^d$;

b. S_0 contains a subset of $S - S^s - S^d$, a non-empty subset of $S^d - S^s \bigcap S^d$, a non-empty subset of $S^s \bigcap S^d$ and nothing in $S^s - S^s \bigcap S^d$;

c. S_0 contains a subset of S- S^s - S^d , a non-empty subset of S^d - $S^s \cap S^d$, a subset of $S^s \cap S^d$ and a non-empty subset of S^s - $S^s \cap S^d$;

Since $S^{c} = (S^{c} - S^{c} \cap S^{d}) \cup (S^{c} \cap S^{d})$, condition *b* and *c* can be combined into:

d. S_0 contains a subset of $S - S^c - S^d$, a non-empty subset of $S^d - S^c \cap S^d$, a non-empty subset of S^c ;

Step 3.1 looks for lightpaths satisfying condition *a*; Step 3.2 does so for condition *d*. As long as there exists a maximum reliable lightpath, the lightpath must be reached in at least one of the two steps following the same proof for (A-2). \blacksquare

The total number of iterations in Step 3.1 is

$$I_{3,l} = 2^{|S-S^s - S^d|} \times (2^{|S^s \cap S^d|} - 1) \times 2^{|S^s - S^s \cap S^d|}$$

= 2^{|S^s| + |S-S^s - S^d|} - 2^{|S-S^d|}

The total number of iterations of Step 3.2 is

$$I_{3,2} = 2^{|S-S^s-S^a|} \times (2^{|S^a-S^s\cap S^a|} - 1) \times (2^{|S^s|} - 1)$$

= 2^{|S|} - 2^{|S^s| + |S-S^s-S^d|} - 2^{|S-S^s|} + 2^{|S-S^s-S^d|}

Thus the total running time of this algorithm is

$$RT_2 = O((2^{|S|} - 2^{|S-S^s|} - 2^{|S-S^d|} + 2^{|S-S^s-S^d|})|N|log|N|)$$
(4)

This is expected for excluding the lightpaths that do not contain fiber links belonging to SRLGs S^s and S^d . Compared to the running time of (OA-1) given in (3), the running time of (OA-2) is clearly smaller. In the special case in which $|S^s| = |S|/2$, $|S^d| = |S|/2$ and $|S^s \cap S^d| = 0$,

$$RT_2 = O((2^{|S|} - 2^{|S|/2} - 2^{|S|/2})|N|log|N|)$$

= $O((2^{|S|} - 2^{1+|S|/2})|N|log|N|)$

The running times of both (OA-1) and (OA-2) are polynomial to the number of network nodes and exponential only to the number of SRLGs, making the algorithms suited for networks that have small numbers of SRLGs (e.g., 20 or less). For networks with arbitrarily large numbers of SRLGs, we propose the following two heuristic solutions.

The first heuristic solution (HA-1) is a modified Dijkstra's Algorithm. In this algorithm, during the process of generating the preferred lightpath P from node s to d, let S^{P} be the set of SRLGs to which the known portion of P belongs, if the SRLG s_{i}^{l} of a fiber link l_{i} is already in S^{P} , we consider l_{i} 's link cost to be zero because adding l_{i} to P does not decrease P's reliability. In this way, new links may be added to P without introducing new SRLG to S^{P} and lowering its reliabilities. The details are as follows.

Algorithm (HA-1)

Step 1: same as Step 1 of (A-1);

- Step 2: initialize set ND to contain all nodes except s;
- Step 3: initialize array CT[] so that CT[v] is the cost of the link from the source to v if v is adjacent to s, and INFINITY otherwise;
- Step 4: initialize entries of array PR[] so that PR[v] is assigned s if v is adjacent to s; NULL otherwise;
- Step 5: initialize array S[] so that S[v] contains the set of SRLGs to which the preferred path from s to v belongs to. Initially S[v] is $\emptyset \forall v \in N$;

Step 6: while
$$(ND \neq \emptyset)$$
 {
choose a node u from ND such that $CT[u]$ is
minimum;
if $(CT[u]$ is INFINITY)

output "no path exists" and quit; delete u from ND;

$$S[u] = S[PR[u]] \cup \{SRLG \text{ of } link < PR[u],$$

u>}; if (u is d) {

$$\begin{cases} for (each node v that is adjacent to u) \\ if (v \in ND) \\ if (the SRLG of link < u, v > \in S[u]) \\ c = CT[u]; \\ else \\ c = CT[u] + cost of link < u, v >; \\ if (c < CT[v]) \\ PR[v] = u; \\ CT[v] = c; \end{cases}$$

This algorithm has the same running time as Dijkstra's algorithm which is O(|N|log|N|).

The next heuristic (HA-2) is an improvement over (HA-

1). The details are as follows:

Algorithm (HA-2) run (HA-1) to find a preferred lightpath P Step 1: *between s and d*; *if (failed) quit;* compute P's reliability r^{P} using (1); $r' = r^{P};$ P' = P;S' = S: s' = null;Step 2: for (every SRLG $s_i \in S'$) { set cost to zero on all links that belong to s_i ; run (HA-1) to find a new path P_i ; if (succeeds) { *compute reliability* r_i^P using (1); if $(r_i^P > r')$ { $r' = r_i^P;$ $P' = P_i$; $s' = s_i;$ restore the link cost that were set to zero; set link cost to zero on all links that belong to s'; while $(s' \neq null)$ { Step 3: S' = S - s'; repeat Step 2; return P' and r';

Step 1 repeats (HA-1). It generates a lightpath which we use as the lower bound to generate more reliable lightpaths in the next two steps. In Step 2, we select one SRLG at a time from all SRLGs in the network, set the link cost to zero on all links that belongs to that particular SRLG, and generate a new lightpath between s and d. The intent is to determine whether we can increase the lightpath reliability if we make the links belonging to one SRLG more attractive than others by setting their cost to zero. After going through all the SRLGs, we may identify an SRLG which leads to the highest reliability of a lightpath. We then set the cost of all the links of that SRLG to zero before going to Step 3. The running time of this step is O(|S||N|log|N|). Step 3 repeats the previous step until no new lightpath can be found with greater reliability. The maximum number of iterations in this step is |S|, which results in a total running time of $O(|S|^2)$ |N|log|N|) for the entire algorithm.

The performances of the algorithms are evaluated by computer simulations in the next section.

IV. SIMULATIONS

We use LEDA programs to randomly generate network graphs for the simulations [23]. The network size ranges from 10 to 40 nodes. The average nodal degree (i.e., the number of links ending in a node) ranges from 2.6 to 3.0. The number of SRLGs in each network ranges from 2 to 10. The non-failure probabilities of the SRLGs are uniformly distributed in the range from 0.9000 to 0.9999. Each fiber link is assumed to support an unlimited number of lightpaths and is assumed to belong to any SRLG with equal probability.

According to the proofs of Section III, (OA-1) and (OA-2) both generate optimal solutions, thus only (OA-1) was executed to provide performance upper bound for the two heuristic solutions (HA-1) and (HA-2). For all pairs of end nodes, we ran (OA-1), (HA-1) and (HA-2) to obtain the maximum reliable lightpaths. We then compared the average reliabilities of the lightpaths. Two sets of the results are depicted in Fig. 2 and Fig. 3. Results from other network topologies are similar.

We observe that the amount of time for the algorithms to generate the maximum reliable lightpaths is very small. For instance, to generate 780 lightpaths between every pair of nodes in a 40-node network with 3.0 nodal degree and 10 SRLGs, it takes less than a second when running the heuristic solutions (HA-1) and (HA-2) on a personal computer with a dual-core Intel processor. Even when running the optimal solution (OA-1), we still get all the paths in less than 30 seconds, i.e., less than 50 milliseconds per lightpath on average. According to (4), (OA-2) should execute even faster than (OA-1).

The simulations demonstrate that the performances of the two heuristic solutions are very close to that of the optimal solution. Heuristic solution (HA-2) in particular, generates lightpaths with reliabilities that are indistinguishable from those of the optimal lightpaths obtained from (OA-1).

We note that the number of SRLGs and their non-failure probabilities have a significant impact on the average reliabilities of the lightpaths. The reliabilities improve along with the increase of the average non-failure probabilities of the SRLGs. In addition, when the number of SRLGs is small, a lightpath is more likely to consist of fiber links that belong to fewer SRLGs. Consequently, from (1), the lightpath has higher reliability if the non-failure probabilities of the SRLGs do not vary significantly. As the number of SRLGs increases, the fiber links of a lightpath are more likely to belong to a bigger variety of SRLGs, which reduces the reliabilities of the lightpaths.

We also note that, as the nodal degree increases, the average reliabilities of the lightpaths increase. The reason for this behavior is that an increase in nodal degree results in higher average number of fiber links belonging to each SRLG, thereby making a lightpath more likely to consist of fiber links that belong to fewer SRLGs.







Figure 3. Average Lightpath Reliability vs. Number of SRLGs. Number of nodes = 40.

The network topology and the size of the network also affect the reliabilities of the lightpath. Larger networks with more nodes result in a higher average hop count for lightpaths; hence, for the same nodal degree and the number of SRLGs, the fiber links of the lightpaths in a network with more nodes will belongs to a greater number of SRLGs than lightpaths in a network with fewer nodes, which reduces the reliabilities of the lightpaths in a larger network.

V. CONCLUSIONS

In this paper we discussed the problem of finding the maximum reliable lightpath between two nodes under multiple failures. For arbitrarily large networks that contain a small number of SRLGs, we proposed two optimal solutions with running times that are polynomial to the network size and exponential to the number of SRLGs. For networks that contain a large number of SRLGs, we proposed two heuristic solutions with running times that are polynomial to both the network size and the number of SRLGs. Computer simulations demonstrated that the heuristics yield solutions that are close to optimal.

From the simulations, we also observed that various factors affect the reliabilities of the lightpaths, including the nodal degree, the number of SRLGs, the non-failure probabilities of individual SRLGs, and the number of nodes in the network. An increase in the nodal degree helps increase the reliabilities of the lightpaths. This is due to the fact that there is a greater choice of routes for the lightpaths. The number of SRLGs also affects the reliabilities of the lightpaths. If the non-failure probabilities of the SRLGs are close to each other, an increase in the number of SRLGs reduces the reliabilities of the lightpaths. Larger number of network nodes also reduces the reliabilities of the lightpaths.

since the lightpaths are now longer and hence are more likely to contain more SRLGs.

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