Math 2401 Final exam review

1. Use the graph of the function $f$ to decide whether the value of the given quantity exists. If it does, find it. If not, explain why.

   $f(1) = \underline{\hspace{2cm}}$.
   $\lim_{x \to 1} f(x) = \underline{\hspace{2cm}}$.
   $f(4) = \underline{\hspace{2cm}}$.
   $\lim_{x \to 4} f(x) = \underline{\hspace{2cm}}$.

2. For the function $f$ whose graph is given, state the value of each quantity, if it exists. If it does not exist, use DNE.

   a. $\lim_{x \to 1} f(x)$
   b. $\lim_{x \to 3^{-}} f(x)$
   c. $\lim_{x \to 3^{+}} f(x)$

3. Find the limit of the function.

   a. $\lim_{x \to 0} \frac{x^4 - 5x^2}{x^2}$.
   b. $\lim_{x \to -1} \frac{x^2 - 2x - 3}{x + 1}$
   c. $\lim_{x \to 1} \frac{x^2 - x}{x^2 - 5x + 4}$
   d. $\lim_{x \to 2} \frac{x^2 - 2x}{x^2 - 4}$

4. Find the derivative of the function.

   a. $f(x) = (3x^2 - 4)(2x + 5)$
   b. $g(s) = \sqrt{s} (s^2 + 8)$
Math 2401 Final exam review

c. \( h(t) = t^3 \cos t \)

d. \( f(x) = \frac{\sqrt{x}}{x^2+1} \)

e. \( g(x) = \frac{\sin x}{x^2} \)

5. Find \( f'(x) \) and find the tangent line of \( f(x) \) at the giving point \( c \).
   a. \( f(x) = (x^2 + 4x)(3x^2 + 2x - 5), \quad c = 0 \)

   b. \( f(x) = x \cos(x), \quad c = \pi \frac{1}{4} \)

   c. \( f(x) = \frac{x-4}{x+4}, \quad c = 3 \)

6. Find the derivative of the trigonometric function.
   a. \( f(t) = t^2 \sin t \)

   b. \( f(\theta) = (\theta^2 + 1) \cos \theta \)
c. $y = \frac{\sec x}{x}$

d. $f(x) = x^2 \tan x$

e. $g(x) = \frac{\sin \theta}{1 - \cos \theta}$

7. Find the second derivative of the function.
   a. $f(x) = x^4 + 2x^3 - 3x^2 - x$

   b. $f(x) = \sec x$

8. Find the derivative of the following functions.
   a. $y = \sqrt{4 - 3x^2}$

   b. $f(x) = \sqrt{x^2 - 4x + 2}$

   c. $y = \sin(2x) \cos(2x)$

   d. $y = x^2 \sec x^2$
e. $y = \sin(\pi x)^2$

f. $h(x) = 5 \tan(3x)$

9. Use implicit differentiation to find $y'$ and evaluate $y'$ at the indicated point.
   a. $y^2 - y - 4x = 0; \quad (0, 1)$
   
   b. $3xy - 2x - 2 = 0; \quad (2, 1)$
   
   c. $2xy + y + 2 = 0; \quad (-1, 2)$

10. For the given function $f(x) = (x - 1)^2 (x + 3)$
    a. Find the critical numbers.
    
    b. Find the open intervals on which the function is increasing or decreasing.
    
    c. Find the inflection points.
d. Find all relative extrema by first derivative test where applicable.

e. Determine the open intervals on which the function is concave upward or concave downward.

11. For the given function \( f(x) = 4 - x - 3x^4 \)
   a. Find the critical numbers.
   b. Find the open intervals on which the function is increasing or decreasing.
   c. Find the inflection points.
   d. Find all relative extrema by first derivative test where applicable.

12. Find the rate of change \( \frac{ds}{dt} \) of the function \( S = x^3 + 4y \) when \( x = 5, y = -6, \frac{dx}{dt} = -1, \) and \( \frac{dy}{dt} = 2. \)
13. A rectangular box has base width twice the size of its base length. The volume of this box is 72 cubic units. Find the dimensions that will minimize surface area.

14. The radius \( r \) of a right circular cone is increasing at a rate of 6 inches per minute, while the height \( h \) of the cone remains constant at 10 inches. Find the rate of change of the volume \( V \) with respect to the time \( t \), when \( r = 12 \) inches. The volume of a right circular cone is \( V = \frac{\pi}{3} r^2 h \).

15. Find all horizontal and vertical asymptotes.
   a. \( f(x) = \frac{x}{x^2-4} \)
   b. \( f(x) = \frac{x^2}{x-3} \)

   Horizontal asymptotes:
   Vertical asymptotes:

16. Find an indefinite integral.
   a. \( \int (1 + 6x)^4 \, dx \)
   b. \( \int x(x^2 - 9)^3 \, dx \)
   c. \( \int t^3 \sqrt{2t^4 + 4} \, dt \)
   d. \( \int \frac{x^2}{(4x^3 - 9)^3} \, dx \)
17. Evaluate the definite integral
   a. \( \int_{0}^{4} \frac{1}{\sqrt{2x+1}} \, dx \)
   b. \( \int_{1}^{2} 2x^2 \sqrt{x^3 + 1} \, dx \)

18. Evaluate the integral using the properties of even and odd functions.
   a. \( \int_{-2}^{2} x^2(x^2 + 1) \, dx \)
   b. \( \int_{-\pi/2}^{\pi/2} \sin x \cos x \, dx \)

19. Differentiate the function.
   a. \( \int_{0}^{x} t^2 \sin t \, dt \)
   b. \( \int_{2}^{x^2} t + 5 \, dt \)

20. Find the area of region \( A1 \) and area of region \( A2 \), between the curves \( y = x + 1 \) and \( y = x^3 - 4x^2 + 5 \)
21. Find the area bounded by the graphs of the equations.

\[ f = \cos^2 x, \quad g = \sin x \cos x, \quad x = -\frac{\pi}{2}, \]  

and \[ x = \frac{\pi}{4}. \]